PRICE INDICES USING AN ARTIFICIAL DATA SET 19

Introduction

19.1 In order to give the reader some idea of how much the various index numbers might differ using a "real" data set, virtually all the major indices defined in the previous chapters are computed in this chapter using an artificial data set consisting of prices and quantities for six commodities over five periods. The data are described in paragraphs 19.3 and 19.4.

19.2 The contents of the remaining sections are outlined in this paragraph. In the section starting with paragraph 19.5, two of the early unweighted indices are computed: the Carli and Jevons indices. Two of the earliest weighted indices are also computed in this section: the Laspeyres and Paasche indices. Both fixed base and chained indices are computed. In the section starting with paragraph 19.9, various asymmetrically[1] weighted indices are computed. In the section starting with paragraph 19.17, symmetrically[2] weighted indices are computed. Some of these indices are superlative, while others are not. The section starting with paragraph 19.23 computes some superlative indices using two stages of aggregation and compares the resulting two-stage indices with their single-stage counterparts. The following section computes various Lloyd–Moulton indices[3] and compares them with superlative indices. The section starting with paragraph 19.32 computes two additive percentage change decompositions for the Fisher ideal index and compares the resulting decompositions, which are found to be very similar. Up to this point, all the indices that are computed are weighted or unweighted *bilateral price indices*; i.e., the index number formula depends only on the price and quantity data pertaining to the two periods whose prices are being compared. In the final three sections of this chapter, various indices involving the data pertaining to three or more periods are computed. In the section starting with paragraph 19.37, Lowe and Young indices are computed where the data of period 1 are used as quantity or share weights in conjunction with the price data of periods 3 to 5, so that the weight reference period is 1 and the price reference period is 3. In the final two sections, various mid-year indices are computed that are based on the Lowe and Young formulae. Recall that for these two index number formulae, the price reference period does not coincide with the weight reference period. Thus these indices are not bilateral index number formulae.

The artificial data set

19.3 The period can be thought of as somewhere between a year and five years. The trends in the data are generally more pronounced than would be seen in the course of a year. The price and quantity data are listed in Tables 19.1 and 19.2. For convenience, the period t nominal expenditures, $p^t q^t \equiv \sum_{i=1}^{n} p_i^t q_i^t$, are listed along with the corresponding period t expenditure shares, $s_i^t \equiv p_i^t q_i^t / p^t q^t$, in Table 19.3.

19.4 The trends that were built into Tables 19.1 to 19.3 are now explained in this paragraph. Think of the first four commodities as the consumption of various classes of *goods* in some economy, while the last two commodities are the consumption of two classes of *services*. Think of the first good as *agricultural consumption*; its quantity fluctuates around 1 and its price also fluctuates around 1.[4] The second good is *energy consumption*; its quantity shows a gently upward trend during the five periods with some minor fluctuations. Note, however, that the price of energy fluctuates wildly from period to period.[5] The third good is *traditional manufactures*. Rather high rates of price inflation are assumed for this commodity for periods 2 and 3 which diminish to a very low inflation rate by the end of the sample period.[6] The consumption of traditional manufactured goods is more or less static in the data set. The fourth commodity is *high-technology manufactured goods*, for example computers, video cameras and compact disks. The demand for these high-technology commodities grows 12 times over the sample period, while the final period price is only one-tenth of the first period price. The fifth commodity is *traditional services*. The price trends for this commodity are similar to those of traditional manufactures, except that the period-to-period

[1] "Asymmetric weights" means that the quantity or value weights for the prices come from only one of the two periods being compared.

[2] "Symmetric weights" means that the quantity or value weights for the prices enter the index number formula in a symmetric or even-handed way.

[3] Recall from Chapter 17 that there is a separate Lloyd–Moulton index for each estimated elasticity of substitution parameter σ that is inserted into the formula.

[4] Note, however, that the expenditure share of agricultural products shows a downward trend over time as the economy develops and shifts into services.

[5] This is an example of the price bouncing phenomenon noted by Szulc (1983). Note that the fluctuations in the price of energy that are built into the data set are not that unrealistic: in the past four years, the price of a barrel of crude oil has fluctuated in the range US$12 to US$40.

[6] This corresponds roughly to the experience of most industrialized countries over the period from 1973 to the mid-1990s. Thus, roughly five years of price movement are compressed into one of our periods.

Table 19.1 Prices for six commodities

Period t	p_1^t	p_2^t	p_3^t	p_4^t	p_5^t	p_6^t
1	1.0	1.0	1.0	1.0	1.0	1.0
2	1.2	3.0	1.3	0.7	1.4	0.8
3	1.0	1.0	1.5	0.5	1.7	0.6
4	0.8	0.5	1.6	0.3	1.9	0.4
5	1.0	1.0	1.6	0.1	2.0	0.2

Table 19.2 Quantities for six commodities

Period t	q_1^t	q_2^t	q_3^t	q_4^t	q_5^t	q_6^t
1	1.0	1.0	2.0	1.0	4.5	0.5
2	0.8	0.9	1.9	1.3	4.7	0.6
3	1.0	1.1	1.8	3.0	5.0	0.8
4	1.2	1.2	1.9	6.0	5.6	1.3
5	0.9	1.2	2.0	12.0	6.5	2.5

Table 19.3 Expenditures and expenditure shares for six commodities

Period t	$p^t q^t$	s_1^t	s_2^t	s_3^t	s_4^t	s_5^t	s_6^t
1	10.00	0.1000	0.1000	0.2000	0.1000	0.4500	0.0500
2	14.10	0.0681	0.1915	0.1752	0.0645	0.4667	0.0340
3	15.28	0.0654	0.0720	0.1767	0.0982	0.5563	0.0314
4	17.56	0.0547	0.0342	0.1731	0.1025	0.6059	0.0296
5	20.00	0.0450	0.0600	0.1600	0.0600	0.6500	0.0250

inflation rates are a little higher. The demand for traditional services, however, grows much more strongly than for traditional manufactures. The final commodity is *high-technology services*, for example telecommunications, wireless phones, Internet services and stock market trading. For this final commodity, the price shows a very strong downward trend to end up at 20 per cent of the starting level, while demand increases fivefold. The movements of prices and quantities in this artificial data set are more pronounced than the year-to-year movements that would be encountered in a typical country, but they do illustrate the problem facing compilers of the consumer price index (CPI); namely, year-to-year price and quantity movements are far from being proportional across commodities, so the choice of index number formula will matter.

Early price indices: The Carli, Jevons, Laspeyres and Paasche indices

19.5 Every price statistician is familiar with the *Laspeyres index* P_L defined by equation (15.5) and the *Paasche index* P_P defined by equation (15.6) in Chapter 15. These indices are listed in Table 19.4 along with two unweighted indices that were considered in previous chapters: the *Carli index* defined by equation (16.45) and the *Jevons index* defined by equation (16.47) in Chapter 16. The indices in Table 19.4 compare the prices in period t with the prices in period 1, that is, they are *fixed base indices*. Thus the period t entry for the Carli index, P_C, is simply the arithmetic mean of the six price relatives, $\sum_{i=1}^{6}(1/6)(p_i^t/p_i^1)$, while the period t entry for the Jevons index, P_J, is the geometric mean of the six price relatives, $\prod_{i=1}^{6}(p_i^t/p_i^1)^{1/6}$.

19.6 Note that by period 5, the spread between the fixed base Laspeyres and Paasche price indices is enormous: P_L is equal to 1.4400 while P_P is 0.7968, a spread of 81 per cent. Since both these indices have exactly the same theoretical justification, it can be seen that the choice of index number formula matters a lot. The period 5 entry for the Carli index, 0.9833, falls between the corresponding Paasche and Laspeyres indices but the period 5 Jevons index, 0.6324, does not. Note that the Jevons index is always considerably below the corresponding Carli index. This will always be the case (unless prices are proportional in the two periods under consideration) because a geometric mean is always equal to or less than the corresponding arithmetic mean.[7]

19.7 It is of interest to recalculate the four indices listed in Table 19.4 using the chain principle rather than the fixed base principle. The expectation is that the spread between the Paasche and Laspeyres indices will be reduced by using the chain principle. These chain indices are listed in Table 19.5.

19.8 It can be seen comparing Tables 19.4 and 19.5 that chaining eliminated about two-thirds of the spread between the Paasche and Laspeyres indices. Nevertheless, even the chained Paasche and Laspeyres indices differ by about 18 per cent in period 5, so the choice of index number formula still matters. Note that chaining did not affect the Jevons index. This is an advantage of the index but the lack of weighting is a fatal flaw.[8] Using the economic approach to index number theory, there is an expectation that the "truth" lies between the Paasche and Laspeyres indices. From Table 19.5, it can be seen that the unweighted Jevons index is far below this acceptable range. Note that chaining did not affect the Carli index in a systematic way for the artificial data set: in periods 3 and 4, the chained Carli is above the corresponding fixed base Carli; but in period 5, the chained Carli is below the fixed base Carli.[9]

Asymmetrically weighted price indices

19.9 This section contains a systematic comparison of all of the asymmetrically weighted price indices (with

[7] According to the theorem of the arithmetic and geometric mean; see Hardy, Littlewood and Pólya (1934, p. 17).

[8] The problem with the evenly weighted geometric mean is that the price declines in high-technology goods and services are given the same weighting as the price changes in the other four commodities (which have rising or stationary price changes), but the expenditure shares of the high-technology commodities remain rather small throughout the five periods. Thus weighted price indices do not show the rate of overall price decrease that the unweighted Jevons index shows. These somewhat negative comments on the use of the unweighted geometric mean as an index number formula at higher levels of aggregation do not preclude its use at the very lowest level of aggregation, where a strong axiomatic justification for the use of this formula can be given. If probability sampling is used at the lowest level of aggregation, then the unweighted geometric mean essentially becomes the logarithmic Laspeyres index.

[9] For many data sets, the chained Carli can be expected to be above the corresponding fixed base Carli; see Szulc (1983).

Table 19.4 The fixed base Laspeyres, Paasche, Carli and Jevons indices

Period t	P_L	P_P	P_C	P_J
1	1.0000	1.0000	1.0000	1.0000
2	1.4200	1.3823	1.4000	1.2419
3	1.3450	1.2031	1.0500	0.9563
4	1.3550	1.0209	0.9167	0.7256
5	1.4400	0.7968	0.9833	0.6324

Table 19.5 Chain Laspeyres, Paasche, Carli and Jevons indices

Period t	P_L	P_P	P_C	P_J
1	1.0000	1.0000	1.0000	1.0000
2	1.4200	1.3823	1.4000	1.2419
3	1.3646	1.2740	1.1664	0.9563
4	1.3351	1.2060	0.9236	0.7256
5	1.3306	1.1234	0.9446	0.6325

Table 19.6 Asymmetrically weighted fixed base indices

Period t	P_{PAL}	P_L	P_{GP}	P_{GL}	P_P	P_{HL}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.6096	1.4200	1.4846	1.3300	1.3824	1.2542
3	1.4161	1.3450	1.3268	1.2523	1.2031	1.1346
4	1.5317	1.3550	1.3282	1.1331	1.0209	0.8732
5	1.6720	1.4400	1.4153	1.0999	0.7968	0.5556

Table 19.7 Asymmetrically weighted indices using the chain principle

Period t	P_{PAL}	P_L	P_{GP}	P_{GL}	P_P	P_{HL}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.6096	1.4200	1.4846	1.3300	1.3824	1.2542
3	1.6927	1.3646	1.4849	1.1578	1.2740	0.9444
4	1.6993	1.3351	1.4531	1.0968	1.2060	0.8586
5	1.7893	1.3306	1.4556	1.0266	1.1234	0.7299

the exception of the Lloyd–Moulton index, which will be considered later). The fixed base indices are listed in Table 19.6. The fixed base Laspeyres and Paasche indices, P_L and P_P, are the same as those indices listed in Table 19.4. The Palgrave index, P_{PAL}, is defined by equation (16.55). The indices denoted by P_{GL} and P_{GP} are the geometric Laspeyres and geometric Paasche indices,[10] which are special cases of the class of geometric indices defined by Konüs and Byushgens (1926); see equation (15.78). For the geometric Laspeyres index, P_{GL}, the exponent weight α_i for the ith price relative is s_i^1, where s_i^1 is the base period expenditure share for commodity i. The resulting index should be considered an alternative to the fixed base Laspeyres index, since both of these indices make use of the same information set. For the geometric Paasche index, P_{GP}, the exponent weight for the ith price relative is s_i^t, where s_i^t is the current period expenditure shares. Finally, the index P_{HL} is the harmonic Laspeyres index that was defined by equation (16.59).

19.10 By looking at the period 5 entries in Table 19.6, it can be seen that the spread between all these fixed base asymmetrically weighted indices has increased to be even larger than the earlier spread of 81 per cent between the fixed base Paasche and Laspeyres indices. In Table 19.6, the period 5 Palgrave index is about three times as big as the period 5 harmonic Laspeyres index, P_{HL}. Again, this illustrates the point that because of the non-proportional growth of prices and quantities in most economies today, the choice of index number formula is very important.

19.11 It is possible to explain why certain of the indices in Table 19.6 are bigger than others. It can be shown that a weighted arithmetic mean of n numbers is equal to or greater than the corresponding weighted geometric mean of the same n numbers, which in turn is equal to or greater than the corresponding weighted harmonic mean of the same n numbers.[11] It can be seen that the three indices P_{PAL}, P_{GP} and P_P all use the current period expenditure shares s_i^t to weight the price relatives (p_i^t/p_i^1), but P_{PAL} is a weighted arithmetic mean of these price relatives, P_{GP} is a weighted geometric mean of these price relatives and P_P is a weighted harmonic mean of these price relatives. Thus by Schlömilch's inequality, it must be the case that:[12]

$$P_{PAL} \geq P_{GP} \geq P_P \qquad (19.1)$$

19.12 Table 19.6 shows that the inequalities (19.1) hold for each period. It can also be verified that the three indices P_L, P_{GL} and P_{HL} all use the base period expenditure shares s_i^1 to weight the price relatives (p_i^t/p_i^1), but P_L is a weighted arithmetic mean of these price relatives, P_{GL} is a weighted geometric mean of these price relatives, and P_{HL} is a weighted harmonic mean of these price relatives. Thus by Schlömilch's inequality, it must be the case that:[13]

$$P_L \geq P_{GL} \geq P_{HL} \qquad (19.2)$$

Table 19.6 shows that the inequalities (19.2) hold for each period.

19.13 All the asymmetrically weighted price indices are compared using the chain principle and are listed in Table 19.7.

19.14 Table 19.7 shows that although the use of the chain principle dramatically reduced the spread between the Paasche and Laspeyres indices P_P and P_L compared to the corresponding fixed base entries in Table 19.6, the spread between the highest and lowest asymmetrically weighted indices in period 5 (the Palgrave index P_{PAL} and the harmonic Laspeyres index P_{HL}) does not fall as much: the fixed base spread is 1.6720/0.5556 = 3.01, while the corresponding chain spread is 1.7893/0.7299 = 2.45.

[10] Vartia (1978, p. 272) used the terms logarithmic Laspeyres and logarithmic Paasche, respectively.

[11] This follows from Schlömilch's (1858) inequality; see Hardy, Littlewood and Pólya (1934, p. 26).

[12] These inequalities were noted by Fisher (1922, p. 92) and Vartia (1978, p. 278).

[13] These inequalities were also noted by Fisher (1922, p. 92) and Vartia (1978, p. 278).

Thus, in this particular case, the use of the chain principle combined with the use of an index number formula that uses the weights of only one of the two periods being compared did not lead to a significant narrowing of the huge differences that these formulae generated using the fixed base principle. With respect to the Paasche and Laspeyres formulae, however, chaining did significantly reduce the spread between these two indices.

19.15 Is there an explanation for the results reported in the previous paragraph? It can be shown that all six of the indices that are found in the inequalities (19.1) and (19.2) approximate each other to the first order around an equal prices and quantities point. Thus with smooth trends in the data, it is expected that all the chain indices will more closely approximate each other than the fixed base indices because the changes in the individual prices and quantities are smaller using the chain principle. This expectation is realized in the case of the Paasche and Laspeyres indices, but not with the others. For some of the commodities in the data set, however, the trends in the prices and quantities are not smooth. In particular, the prices for the first two commodities (agricultural products and oil) bounce up and down. As noted by Szulc (1983), this will tend to cause the chain indices to have a wider dispersion than their fixed base counterparts. In order to determine if it is the bouncing prices problem that is causing some of the chained indices in Table 19.7 to diverge from their fixed base counterparts, all the indices in Tables 19.6 and 19.7 were computed again but excluding commodities 1 and 2 from the computations. The results of excluding these bouncing commodities may be found in Tables 19.8 and 19.9.

19.16 It can be seen that excluding the bouncing price commodities does cause the chain indices to have a much narrower spread than their fixed base counterparts. Thus, the conclusion is that if the underlying price and quantity data are subject to reasonably smooth trends over time, then the use of chain indices will narrow considerably the dispersion in the asymmetrically weighted indices. In the next section, index number formulae that use weights from both periods in a symmetric or even-handed manner are computed.

Table 19.8 Asymmetrically weighted fixed base indices for commodities 3–6

Period t	P_{PAL}	P_L	P_{GP}	P_{GL}	P_P	P_{HL}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2877	1.2500	1.2621	1.2169	1.2282	1.1754
3	1.4824	1.4313	1.3879	1.3248	1.2434	1.1741
4	1.6143	1.5312	1.4204	1.3110	1.0811	0.9754
5	1.7508	1.5500	1.4742	1.1264	0.7783	0.5000

Table 19.9 Asymmetrically weighted chained indices for commodities 3–6

Period t	P_{PAL}	P_L	P_{GP}	P_{GL}	P_P	P_{HL}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2877	1.2500	1.2621	1.2169	1.2282	1.1754
3	1.4527	1.4188	1.4029	1.3634	1.3401	1.2953
4	1.5036	1.4640	1.4249	1.3799	1.3276	1.2782
5	1.4729	1.3817	1.3477	1.2337	1.1794	1.0440

Symmetrically weighted indices: Superlative and other indices

19.17 Symmetrically weighted indices can be decomposed into two classes: *superlative indices* and *other symmetrically weighted indices*. Superlative indices have a close connection to economic theory. As was seen in paragraphs 17.27 to 17.49 of Chapter 17, a superlative index is exact for a representation of the consumer's preference function or the dual unit cost function that can provide a second-order approximation to arbitrary (homothetic) preferences. Four important superlative indices were considered in previous chapters:

- the *Fisher ideal price index* P_F, defined by equation (15.12);
- the *Walsh price index* P_W, defined by equation (15.19) (this price index also corresponds to the quantity index Q^1, defined by equation (17.33) in Chapter 17);
- the *Törnqvist–Theil price index* P_T, defined by equation (15.81);
- the *implicit Walsh price index* P_{IW} that corresponds to the Walsh quantity index Q_W defined in Chapter 15 (this is also the index P^1 defined by equation (17.38)).

19.18 These four symmetrically weighted superlative price indices are listed in Table 19.10 using the fixed base principle. Also listed in Table 19.10 are two symmetrically weighted (but not superlative) price indices:[14]

- *the Marshall–Edgeworth price index* P_{ME}, defined in paragraph 15.18;
- *the Drobisch price index* P_D, defined by equation (15.12).

19.19 Note that the Drobisch index P_D is always equal to or greater than the corresponding Fisher index P_F. This follows from the fact that the Fisher index is the geometric mean of the Paasche and Laspeyres indices, while the Drobisch index is the arithmetic mean of the Paasche and Laspeyres indices, and an arithmetic mean is always equal to or greater than the corresponding geometric mean. Comparing the fixed base asymmetrically weighted indices in Table 19.6 with the symmetrically weighted indices in Table 19.10, it can be seen that the spread between the lowest and highest index in period 5 is much less for the symmetrically weighted indices. The spread is $1.6720/0.5556 = 3.01$ for the asymmetrically weighted indices, but only $1.2477/0.9801 = 1.27$ for the symmetrically weighted indices. If the comparisons are restricted to the superlative indices listed for period 5 in Table 19.10, then this spread is further reduced to $1.2477/1.0712 = 1.16$; i.e., the spread between the fixed base superlative indices is "only" 16 per cent compared to the fixed base spread between the Paasche and Laspeyres indices of 81 per cent ($1.4400/0.7968 = 1.81$). There is an expectation that the spread between the superlative indices will be further reduced by using the chain principle.

[14] Diewert (1978, p. 897) showed that the Drobisch–Sidgwick–Bowley price index approximates any superlative index to the second order around an equal price and quantity point; i.e., P_{SB} is a *pseudo-superlative index*. Straightforward computations show that the Marshall–Edgeworth index P_{ME} is also pseudo-superlative.

Table 19.10 Symmetrically weighted fixed base indices

Period t	P_T	P_{IW}	P_W	P_F	P_D	P_{ME}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4052	1.4015	1.4017	1.4011	1.4012	1.4010
3	1.2890	1.2854	1.2850	1.2721	1.2741	1.2656
4	1.2268	1.2174	1.2193	1.1762	1.1880	1.1438
5	1.2477	1.2206	1.1850	1.0712	1.1184	0.9801

Table 19.11 Symmetrically weighted indices using the chain principle

Period t	P_T	P_{IW}	P_W	P_F	P_D	P_{ME}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4052	1.4015	1.4017	1.4011	1.4012	1.4010
3	1.3112	1.3203	1.3207	1.3185	1.3193	1.3165
4	1.2624	1.2723	1.2731	1.2689	1.2706	1.2651
5	1.2224	1.2333	1.2304	1.2226	1.2270	1.2155

19.20 The symmetrically weighted indices are computed using the chain principle. The results may be found in Table 19.11.

19.21 A quick glance at Table 19.11 shows that *the combined effect of using both the chain principle as well as symmetrically weighted indices is to dramatically reduce the spread between all indices constructed using these two principles*. The spread between all the symmetrically weighted indices in period 5 is only 1.2333/1.2155 = 1.015 or 1.5 per cent and the spread between the four superlative indices in period 5 is an even smaller 1.2333/1.2224 = 1.009, or about 0.1 per cent. The spread in period 5 between the two most commonly used superlative indices, the Fisher P_F and the Törnqvist P_T, is truly tiny: 1.2226/1.2224 = 0.0002.[15]

19.22 The results listed in Table 19.11 reinforce the numerical results tabled by Hill (2002) and Diewert (1978, p. 894); *the most commonly used chained superlative indices will generally give approximately the same numerical results*.[16] In particular, the chained Fisher, Törnqvist and Walsh indices will generally approximate each other very closely.

Superlative indices constructed in two stages of aggregation

19.23 Attention is now directed to the differences between superlative indices and their counterparts that are constructed in two stages of aggregation; see paragraphs 17.55 to 17.60 of Chapter 17 for a discussion of the issues and a listing of the formulae used. Using the artificial data set, the first four commodities are combined into a *goods aggregate* and the last two commodities into a *services aggregate*. In the second stage of aggregation, the goods and services components will be aggregated into an all-items index.

19.24 The results for the two-stage aggregation procedure using period 1 as the fixed base for the Fisher index P_F, the Törnqvist index P_T and the Walsh and implicit Walsh indexes, P_W and P_{IW}, are reported in Table 19.12.

19.25 Table 19.12 shows that the fixed base single stage superlative indices generally approximate their fixed base two-stage counterparts fairly closely, with the exception of the Fisher formula. The divergence between the single-stage Fisher index P_F and its two-stage counterpart P_{F2S} in period 5 is 1.1286/1.0712 = 1.05 or 5 per cent. The other divergences are 2 per cent or less.

19.26 Using chain indices, the results of the two-stage aggregation procedure are reported in Table 19.13. Again, the single-stage and their two-stage counterparts are listed for the Fisher index P_F, the Törnqvist index P_T and the Walsh and implicit Walsh indexes, P_W and P_{IW}.

19.27 Table 19.13 shows that the chained single-stage superlative indices generally approximate their fixed base two-stage counterparts very closely indeed. The divergence between the chained single-stage Törnqvist index P_T and its two-stage counterpart P_{T2S} in period 5 is 1.2300/1.2224 = 1.006 or 0.6 per cent. The other divergences are all less than this. Given the large dispersion in period-to-period price movements, these two-stage aggregation errors are not large.

Lloyd–Moulton price indices

19.28 The next formula that will be illustrated using the artificial data set is the Lloyd (1975) and Moulton (1996) index P_{LM}, defined by equation (17.71). Recall that this formula requires an estimate for the parameter σ, *the elasticity of substitution* between all commodities being aggregated. Recall also that if σ equals 0, then the Lloyd–Moulton index collapses down to the ordinary Laspeyres index, P_L. When σ equals 1, the Lloyd–Moulton index is not defined, but it can be shown that the limit of $P_{LM\sigma}$ as σ approaches 1 is P_{GL}, the geometric Laspeyres index or the logarithmic Laspeyres index with base period shares as weights. This index uses the same basic information as the fixed base Laspeyres index P_L, and so it is a possible alternative index for CPI compilers to use. As was shown by Shapiro and Wilcox (1997a),[17] the Lloyd–Moulton index may be used to approximate a superlative index using the same information that is used in the construction of a fixed base Laspeyres index, provided that an estimate for the parameter σ is available. This methodology will be tested using the artificial data set. The superlative index that is to be approximated is the chain Fisher index[18] (which approximates the other

[15] In other periods, the differences were nevertheless larger. On average over the last four periods, the chain Fisher and the chain Törnqvist indices differed by 0.0025 percentage points.

[16] More precisely, the superlative quadratic mean of order r price indices P^r defined by equation (17.35) and the implicit quadratic mean of order r price indices P^{r*} defined by equation (17.32) will generally closely approximate each other, provided that r is in the interval $0 \leq r \leq 2$.

[17] Alterman, Diewert and Feenstra (1999) also used this methodology in the context of estimating superlative international trade price indices.

[18] Since there is still a considerable amount of dispersion among the fixed base superlative indices and practically no dispersion between the chained superlative indices, the Fisher chain index is taken as the target index rather than any of the fixed base superlative indices.

Table 19.12 Fixed base superlative single-stage and two-stage indices

Period t	P_F	P_{F2S}	P_T	P_{T2S}	P_W	P_{W2S}	P_{IW}	P_{IW2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4011	1.4004	1.4052	1.4052	1.4017	1.4015	1.4015	1.4022
3	1.2721	1.2789	1.2890	1.2872	1.2850	1.2868	1.2854	1.2862
4	1.1762	1.2019	1.2268	1.2243	1.2193	1.2253	1.2174	1.2209
5	1.0712	1.1286	1.2477	1.2441	1.1850	1.2075	1.2206	1.2240

Table 19.13 Chained superlative single-stage and two-stage indices

Period t	P_F	P_{F2S}	P_T	P_{T2S}	P_W	P_{W2S}	P_{IW}	P_{IW2S}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4011	1.4004	1.4052	1.4052	1.4017	1.4015	1.4015	1.4022
3	1.3185	1.3200	1.3112	1.3168	1.3207	1.3202	1.3203	1.3201
4	1.2689	1.2716	1.2624	1.2683	1.2731	1.2728	1.2723	1.2720
5	1.2226	1.2267	1.2224	1.2300	1.2304	1.2313	1.2333	1.2330

Table 19.14 Chained Fisher and fixed base Lloyd–Moulton indices

Period t	P_F	P_{LM0}	$P_{LM.2}$	$P_{LM.3}$	$P_{LM.4}$	$P_{LM.5}$	$P_{LM.6}$	$P_{LM.7}$	$P_{LM.8}$	P_{LM1}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4011	1.4200	1.4005	1.3910	1.3818	1.3727	1.3638	1.3551	1.3466	1.3300
3	1.3185	1.3450	1.3287	1.3201	1.3113	1.3021	1.2927	1.2831	1.2731	1.2523
4	1.2689	1.3550	1.3172	1.2970	1.2759	1.2540	1.2312	1.2077	1.1835	1.1331
5	1.2226	1.4400	1.3940	1.3678	1.3389	1.3073	1.2726	1.2346	1.1932	1.0999

chained superlative indices listed in Table 19.11 very closely). The chained Fisher index P_F is listed in column 2 of Table 19.14 along with the fixed base Lloyd–Moulton indices $P_{LM\sigma}$ for σ equal to 0 (this reduces to the fixed base Laspeyres index P_L), 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 1 (which is the fixed base geometric index P_{GL}). Note that the Lloyd–Moulton indices steadily decrease as the elasticity of substitution σ is increased.[19]

19.29 Table 19.14 shows that no single choice of the elasticity of substitution σ will lead to a Lloyd–Moulton price index $P_{LM\sigma}$ that will closely approximate the chained Fisher index P_F for periods 2, 3, 4 and 5. To approximate P_F in period 2, it is necessary to choose σ close to 0.1; to approximate P_F in period 3, choose σ close to 0.3; to approximate P_F in period 4, choose σ between 0.4 and 0.5; and to approximate P_F in period 5, choose σ between 0.7 and 0.8.[20]

19.30 The computations for the Lloyd–Moulton indices listed in Table 19.14 are now repeated except that the chain principle is used to construct the Lloyd–Moulton indices; see Table 19.15. Again, the object is to approximate the chained Fisher price index P_F which is listed as the second column in Table 19.15. In Table 19.15, P_{LM0} is the chained Laspeyres index and P_{LM1} is the chained geometric Laspeyres or geometric index using the expenditure shares of the previous period as weights.

19.31 Table 19.15 shows that again no single choice of the elasticity of substitution σ will lead to a Lloyd–Moulton price index $P_{LM\sigma}$ that will closely approximate the chained Fisher index P_F for all periods. To approximate P_F in period 2, choose σ close to 0.1; to approximate P_F in period 3, choose σ close to 0.2; to approximate P_F in period 4, choose σ between 0.2 and 0.3; and to approximate P_F in period 5, choose σ between 0.3 and 0.4. It should be noted, however, that if σ is chosen to equal to 0.3 and the resulting chained Lloyd–Moulton index $P_{LM.3}$ is used to approximate the chained Fisher index P_F, then a much better approximation to P_F results than that provided by either the chained Laspeyres index (see P_{LM0} in the third column of Table 19.15) or the fixed base Laspeyres index (see P_{LM0} in the third column of Table 19.14).[21] The tentative conclusions on the use of the Lloyd–Moulton index to approximate superlative indices that can be drawn from the above tables are:

- the elasticity of substitution parameter σ which appears in the Lloyd–Moulton formula is unlikely to remain constant over time, and hence it will be necessary for statistical agencies to update their estimates of σ at regular intervals;
- the use of the Lloyd–Moulton index as a real-time preliminary estimator for a chained superlative index

[19] This follows from Schlömilch's (1858) inequality again.

[20] Unfortunately, for this data set, neither the fixed base Laspeyres index $P_L = P_{LM0}$ nor the fixed base weighted geometric index $P_{GL} = P_{LM1}$ is very close to the chain Fisher index for all periods. For less extreme data sets, the fixed base Laspeyres and fixed base geometric indices will be closer to the chained Fisher index.

[21] For this particular data set, the fixed base or chained geometric indices using either the expenditure weights of period 1 (see the last column of Table 19.14) or using the weights of the previous period (see the last column of Table 19.15) do not approximate the chained Fisher index very closely. For less extreme data sets, however, the use of chained Laspeyres or geometric indices may approximate a chained superlative index adequately.

Table 19.15 Chained Fisher and chained Lloyd–Moulton indices

Period t	P_F	P_{LM0}	$P_{LM.2}$	$P_{LM.3}$	$P_{LM.4}$	$P_{LM.5}$	$P_{LM.6}$	$P_{LM.7}$	$P_{LM.8}$	P_{LM1}
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4011	1.4200	1.4005	1.3910	1.3818	1.3727	1.3638	1.3551	1.3466	1.3300
3	1.3185	1.3646	1.3242	1.3039	1.2834	1.2628	1.2421	1.2212	1.2002	1.1578
4	1.2689	1.3351	1.2882	1.2646	1.2409	1.2171	1.1932	1.1692	1.1452	1.0968
5	1.2226	1.3306	1.2702	1.2400	1.2097	1.1793	1.1488	1.1183	1.0878	1.0266

Table 19.16 Diewert's additive percentage change decomposition of the Fisher index

Period t	$P_F - 1$	$v_{F1}\Delta p_1$	$v_{F2}\Delta p_2$	$v_{F3}\Delta p_3$	$v_{F4}\Delta p_4$	$v_{F5}\Delta p_5$	$v_{F6}\Delta p_6$
2	0.4011	0.0176	0.1877	0.0580	−0.0351	0.1840	−0.0111
3	−0.0589	−0.0118	−0.1315	0.0246	−0.0274	0.0963	−0.0092
4	−0.0376	−0.0131	−0.0345	0.0111	−0.0523	0.0635	−0.0123
5	−0.0365	0.0112	0.0316	0.0000	−0.0915	0.0316	−0.0194

seems warranted, provided that the statistical agency can provide estimates for chained superlative indices on a delayed basis. The Lloyd–Moulton index would provide a useful supplement to the traditional fixed base Laspeyres price index.

Additive percentage change decompositions for the Fisher ideal index

19.32 The next formulae to be illustrated using the artificial data set are the *additive percentage change decompositions* for the Fisher ideal index, discussed in paragraphs 16.62 to 16.73 of Chapter 16.[22] The chain links for the Fisher price index are first decomposed into additive components using the formulae (16.38) to (16.40). The results of the decomposition are listed in Table 19.16. Thus $P_F - 1$ is the percentage change in the Fisher ideal chain link going from period $t-1$ to t, and the decomposition factor $v_{Fi}\Delta P_i = v_{Fi}(P_i^t - p_i^{t-1})$ is the contribution to the total percentage change of the change in the ith price from p_i^{t-1} to p_i^t for $i = 1, 2, \ldots, 6$.

19.33 Table 19.16 shows that the price index going from period 1 to 2 grew about 40 per cent, and the major contributors to this change were the increases in the price of commodity 2, energy (18.77 per cent), and in commodity 5, traditional services (18.4 per cent). The increase in the price of traditional manufactured goods, commodity 3, contributed 5.8 per cent to the overall increase of 40.11 per cent. The decreases in the prices of high-technology goods (commodity 4) and high-technology services (commodity 6) offset the other increases by −3.51 per cent and −1.11 per cent going from period 1 to 2. Going from period 2 to 3, the overall change in prices was negative: −5.89 per cent. The reader can read across row 3 of Table 19.16 to see what was the contribution of the six component price changes to the overall price change. It is evident that a big price change in a particular component i, combined with a big expenditure share in the two periods under consideration will lead to a big decomposition factor, v_{Fi}.

19.34 The next set of computations to be illustrated using the artificial data set is the additive percentage change decomposition for the Fisher ideal index according to Van Ijzeren (1987, p. 6), which was mentioned in footnote 43 of Chapter 16.[23] The price counterpart to the additive decomposition for a quantity index is:

$$P_F(p^0, p^1, q^0, q^1) = \frac{\sum_{i=1}^{n} q_{Fi}^* p_i^1}{\sum_{i=1}^{n} q_{Fi}^* p_i^0} \tag{19.3}$$

where the reference quantities need to be defined somehow. Van Ijzeren (1987, p. 6) showed that the following reference weights provide an exact additive representation for the Fisher ideal price index:

$$q_{Fi}^* \equiv (1/2)q_i^0 + \{(1/2)q_i^1/Q_F(p^0, p^1, q^0, q^1)\} \quad \text{for } i = 1, 2, \ldots, 6 \tag{19.4}$$

where Q_F is the overall Fisher quantity index. Thus using the Van Ijzeren quantity weights q_{Fi}^*, the following Van Ijzeren additive percentage change decomposition for the Fisher price index is obtained:

$$P_F(p^0, p^1, q^0, q^1) - 1 = \left\{ \sum_{i=1}^{6} q_{Fi}^* p_i^1 \middle/ \sum_{i=1}^{6} q_{Fi}^* p_i^0 \right\} - 1 = \sum_{i=1}^{6} v_{Fi}^* \{ p_i^1 - p_i^0 \} \tag{19.5}$$

where the Van Ijzeren weight for commodity i, v_{Fi}^*, is defined as

$$v_{Fi}^* \equiv q_{Fi}^* \Big/ \sum_{i=1}^{6} q_{Fi}^* p_i^0 \quad \text{for } i = 1, 2, \ldots, 6 \tag{19.6}$$

[22] See Diewert (2002a, p. 73).

[23] See Reinsdorf, Diewert and Ehemann (2002) for additional information on this decomposition.

Table 19.17 Van Ijzeren's decomposition of the Fisher price index

Period t	$P_F - 1$	$v^*_{F1}\Delta p_1$	$v^*_{F2}\Delta p_2$	$v^*_{F3}\Delta p_3$	$v^*_{F4}\Delta p_4$	$v^*_{F5}\Delta p_5$	$v^*_{F6}\Delta p_6$
2	0.4011	0.0178	0.1882	0.0579	−0.0341	0.1822	−0.0109
3	−0.0589	−0.0117	−0.1302	0.0243	−0.0274	0.0952	−0.0091
4	−0.0376	−0.0130	−0.0342	0.0110	−0.0521	0.0629	−0.0123
5	−0.0365	0.0110	0.0310	0.0000	−0.0904	0.0311	−0.0191

Table 19.18 The Lowe and Young indices, the fixed base Laspeyres, Paasche and Fisher indices, and the chained Laspeyres, Paasche and Fisher indices

Period t	P_{Lo}	P_Y	P_L	P_P	P_F	P_{LCH}	P_{PCH}	P_{FCH}
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	1.0074	0.9396	0.9784	0.9466	0.9624	0.9784	0.9466	0.9624
5	1.0706	0.9794	1.0105	0.8457	0.9244	0.9751	0.8818	0.9273

19.35 The chain links for the Fisher price index will be decomposed into price change components using the formulae (19.4) to (19.6), listed above. The results of the decomposition are listed in Table 19.17. Thus $P_F - 1$ is the percentage change in the Fisher ideal chain link going from period $t-1$ to t and the Van Ijzeren decomposition factor $v^*_{Fi}\Delta p_i$ is the contribution to the total percentage change of the change in the ith price from p_i^{t-1} to for $i=1,2,\ldots,6$.

19.36 Comparing the entries in Tables 19.16 and 19.17, it can be seen that the differences between the Diewert and Van Ijzeren decompositions of the Fisher price index are very small. The maximum absolute difference between the $v_{Fi}\Delta p_i$ and $v^*_{Fi}\Delta p_i$ is only 0.0018 (about 0.2 percentage points) and the average absolute difference is 0.0003. This is somewhat surprising given the very different nature of the two decompositions.[24] As was mentioned in footnote 43 of Chapter 16, the Van Ijzeren decomposition of the chain Fisher quantity index is used by the Bureau of Economic Analysis in the United States.[25]

The Lowe and Young indices

19.37 Recall that the Lowe index was defined by equation (15.15) in Chapter 15. If it is desired to compare the prices in period t with those in period 0, the formula for the Lowe index is given by equation (19.7) below:

$$P_{Lo}(p^1, p^t, q^b) \equiv \sum_{i=1}^{6} p_i^t q_i^b \Big/ \sum_{i=1}^{6} p_i^0 q_i^b \quad t=1,2,\ldots,5 \tag{19.7}$$

where $q^b \equiv [q_1^b, q_2^b, \ldots, q_6^b]$ is the quantity vector pertaining to a base period b which is prior to period 0, the base period for prices. This index will be computed for periods t equal to 3 to 5 for the artificial data set where the quantity reference period b is taken to be period 1 and the price reference period 0 is taken to be period 3; see the column with the heading P_{Lo} in Table 19.18.

19.38 For comparison purposes, the fixed base Laspeyres, Paasche and Fisher indices are also calculated for periods 3, 4 and 5, where period 3 is treated as the base period; see the columns with the headings P_L, P_P and P_F respectively. The chained Laspeyres, Paasche and Fisher indices are also calculated for periods 3, 4 and 5, and listed in Table 19.18; see the columns with the headings P_{LCH}, P_{PCH} and P_{FCH} respectively. Table 19.18 shows that the Lowe index is higher than all six of these comparison indices in periods 4 and 5. In particular, the Lowe index P_{Lo} is greater than the fixed base Laspeyres index P_L for periods 4 and 5, which is consistent with the inequality (15.37) in Chapter 15, where it was argued that the Lowe index would exceed the Laspeyres index if there were long-run trends in prices. Compared with the preferred fixed base or chained Fisher ideal target indices, P_F or P_{FCH}, the Lowe index has a considerable upward bias for this trending artificial data set.

19.39 The Young index was defined by equation (15.48) in Chapter 15 and, for convenience, this definition is repeated below:

$$P_Y(p^0, p^t, s^b) \equiv \sum_{i=1}^{n} s_i^b (p_i^t / p_i^0) \tag{19.8}$$

The base period b expenditure shares for the commodities are the s_i^b in equation (19.8) and the price reference period is period 0. This Young index will be computed for periods t equal to 3 to 5 for the artificial data set, where the quantity reference period b is taken to be period 1 and the price reference period 0 is taken to be period 3; see the column with the heading P_Y in Table 19.18.

19.40 For periods 4 and 5, the Young index is below the corresponding values for the fixed base Laspeyres index.[26] For period 4, the Young index is 0.9396, which is below the corresponding value for the Fisher index, which is 0.9624. However, for period 5, the Young index is 0.9794, which is above the corresponding values for

[24] Reinsdorf, Diewert and Ehemann (2002) nevertheless show that the terms in the two decompositions approximate each other to the second order around any point where the two price vectors are equal and where the two quantity vectors are equal.

[25] See Moulton and Seskin (1999), and Ehemann, Katz and Moulton (2002).

[26] It is noted in Chapter 15 that the Young index can be above or below the corresponding fixed base Laspeyres index, depending on the responsiveness of expenditure shares to changes in prices.

the two target Fisher indices, which are 0.9244 for the fixed base index and 0.9273 for the chained index. Thus, although the direction of the bias in the Young index is not always the same, it can be seen that it has substantial biases for the artificial data set compared to the preferred target indices.

Mid-year indices based on the Lowe formula

19.41 Recall the Lowe index formula (19.7). In most applications of the formula by statistical agencies, the quantity vector q will be taken from a period that is prior to the base period for prices, which is period 1 in the artificial data set. It is also possible, however, to use the formula as a type of mid-year index, where the reference quantity vector q could be taken to be an average of the quantity vectors pertaining to periods 1 to 5. This possible use of the formula will be explored in the present section. Thus the first Lowe index, P_{Lo1}, sets q in formula (19.7) equal to q^1, the period 1 quantity vector in the artificial data set. This index turns out to be identical to the fixed base Laspeyres index P_L, which was listed earlier in Table 19.4. The second Lowe index, P_{Lo2}, sets q in formula (19.7) equal to the average of the period 1 and 2 quantity vectors, $(1/2)(q^1+q^2)$.[27] The third Lowe index, P_{Lo3}, sets q equal to the average of the period 1 to 3 quantity vectors, $(1/3)(q^1+q^2+q^3)$. The fourth Lowe index, P_{Lo4}, sets q equal to the average of the period 1 to 4 quantity vectors, $(1/4)(q^1+q^2+q^3+q^4)$. Finally, the fifth Lowe index, P_{Lo5}, sets q equal to the average of the period 1 to 5 quantity vectors, $(1/5)(q^1+q^2+q^3+q^4+q^5)$.[28] The resulting five Lowe type indices are listed in Table 19.19.

19.42 The mid-year index $P_{MY} \equiv P_{Lo}(p^1, p^t, q^3)$ was defined in paragraphs 15.49 to 15.53 of Chapter 15; it is a Lowe type index with the "representative" quantity vector q chosen to be q^3, the quantity vector that pertains to the middle period in the span of periods under consideration (which is periods 1 to 5 in the numerical example). It is listed as the seventh column in Table 19.18.[29] The mid-year index and the five Lowe indices are compared to two of the "best" target indices, the chain Törnqvist and chain Fisher indices, P_T and P_F listed in the last two columns of Table 19.19.

19.43 From Table 19.19, it can be seen that none of the Lowe type indices (or the mid-year index) are very close to the two target indices (the chain Törnqvist and chain Fisher) for all periods.[30] With less extreme data sets, however, it is quite possible that the fifth Lowe index and the mid-year index could form adequate approximations to the target indices.

19.44 With strong trends in the price data and normal consumer substitution responses, it is unlikely that Lowe type indices, based on averages of the quantity data pertaining to the first few periods in a long time series of data, will be able to provide an adequate approximation to a chained superlative index. In general, this type of Lowe index will suffer from an upward bias compared to the target index, as can be seen from Table 19.19.

Young-type indices

19.45 Recall that the Young index was defined by equation (15.48) in Chapter 15, or equation (19.8) above. If it is desired to compare the prices in period t with those in period 1, the formula for the Young index is given by equation (19.9):

$$P_Y(p^1, p^t, s^b) \equiv \sum_{i=1}^{6} s_i^b (p_i^t / p_i^1) \quad \text{for } t = 1, 2, \ldots, 5 \quad (19.9)$$

where the expenditure share vector $s^b \equiv [s_1^b, \ldots s_6^b]$ is "representative" for the span of periods under consideration. In most applications of the formula by statistical agencies, the base period expenditure share vector s^b will be taken from a period that is prior to the base period for prices, which is period 1 in the artificial data set. For illustrative purposes, rather than adding new data to the artificial data set, the reference share vector s^b will be taken to be an average of the expenditure share vectors pertaining to periods 1 to 5. Thus, the first Young-type index, P_{Y1}, sets s^b in formula (19.9) equal to s^1, the period 1 expenditure share vector in the artificial data set. This index turns out to be identical to the fixed base Laspeyres index P_L, shown in Table 19.4. The second Young-type index, P_{Y2}, sets s^b in formula (19.9) equal to the average of the period 1 and 2 share vectors, $(1/2)(s^1+s^2)$. The third Young-type index, P_{Y3}, sets s^b equal to the average of the period 1 to 3 share vectors, $(1/3)(s^1+s^2+s^3)$. The fourth Young-type index, P_{Y4}, sets s^b equal to the average of the period 1 to 4 share vectors, $(1/4)(s^1+s^2+s^3+s^4)$. Finally, the fifth Young-type index, P_{Y5}, sets s^b equal to the average of the period 1 to 5 share vectors, $(1/5)(s^1+s^2+s^3+s^4+s^5)$. The resulting five Young type indices are listed in Table 19.20 below. These indices are compared to two of the "best" target indices, the chained Törnqvist and chained Fisher indices, P_T and P_F listed in the last two columns of Table 19.20.

19.46 Table 19.20 shows that all the Young-type indices exhibit a substantial upward bias compared to the target chain Törnqvist and Fisher indices, P_T and P_F. Comparing Table 19.19 with Table 19.20, it can be seen that the bias in the Young-type indices becomes bigger as the expenditure shares become more representative of all five periods, whereas the upward bias in the Lowe-type indices tends to become smaller as the reference quantity vector became more representative of all periods.

[27] This is the Lowe index for the artificial data set, which will probably be the most comparable to the type of Lowe index currently computed by statistical agencies.

[28] This is Walsh's (1901, p. 431) multi-year fixed basket index, where the quantity vector is chosen to be the arithmetic average of the quantity vectors in the time period under consideration.

[29] It can be verified that if there are exact linear time trends in the quantity data, then the mid-year index P_{MY} will be exactly equal to the fifth Lowe index, P_{Lo5}.

[30] The fourth Lowe index P_{Lo4} and the mid-year index P_{MY} appear to be the closest to the target indices.

Table 19.19 The five Lowe indices, the mid-year index, and the Törnqvist and Fisher chain indices

Period t	P_{Lo1}	P_{Lo2}	P_{Lo3}	P_{Lo4}	P_{Lo5}	P_{MY}	P_T	P_F
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4200	1.4010	1.3641	1.3068	1.2267	1.3055	1.4052	1.4011
3	1.3450	1.3366	1.2851	1.2142	1.1234	1.2031	1.3112	1.3185
4	1.3550	1.3485	1.2824	1.1926	1.0801	1.1772	1.2624	1.2689
5	1.4400	1.4252	1.3444	1.2321	1.0868	1.2157	1.2224	1.2226

Table 19.20 The five Young-type indices and the Törnqvist and Fisher chain indices

Period t	P_{Y1}	P_{Y2}	P_{Y3}	P_{Y4}	P_{Y5}	P_T	P_F
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4200	1.5148	1.4755	1.4409	1.4355	1.4052	1.4011
3	1.3450	1.3567	1.3765	1.3943	1.4144	1.3112	1.3185
4	1.3550	1.3526	1.3917	1.4267	1.4584	1.2624	1.2689
5	1.4400	1.4632	1.4918	1.5173	1.5482	1.2224	1.2226

19.47 Note that the Young type indices P_{Y2} to P_{Y5} are all bigger in magnitude than P_{Y1}, which is the ordinary fixed base Laspeyres index. It must be recognized, however, that these Young type indices are not the type of Young index that is computed by statistical agencies, in which the weight reference period precedes the price reference period. As discussed in paragraphs 19.39 to 19.42, this latter type of Young index could be above or below the corresponding fixed base Laspeyres index.

19.48 The results of this section and the previous one can be summarized as follows: it appears to be a useful exercise to attempt to find quantity weights for the Lowe formula that are representative for the entire period covered by the index, but it does not appear to be useful to do the same for the Young formula.