QUALITY CHANGE AND HEDONICS 21

Introduction

21.1 Chapters 15 to 20 cover theoretical issues relating to the choice of index number formula and are based on a simplifying assumption: that the aggregation is over the same $i = 1, \ldots n$ matched items in the two periods being compared. A comparison of prices between two periods requires the quality of each item to remain the same between the periods. Price collectors are asked to match items with the same quality specification in each month, so that only ''pure'' price changes are measured, not price changes tainted by changes in the quality of what is consumed. In practice, the quality of what is consumed does change. Furthermore, new goods and services appear on the market, and their relative price changes may differ from the price changes of existing ones. In addition, the expenditure share of these new goods and services may be substantial. Paragraphs 21.2 to 21.60 outline a theoretical framework which extends the definition of items to include their quality characteristics. It helps to provide a background for the practical implementation of quality adjustment, discussed in Chapter 7, and for ways of dealing with item substitution and new goods, covered in Chapter 8.

New and disappearing items

21.2 The assumption in the previous chapters was that the same set of items was being compared in each period. This can be considered as sampling from all the matched items available in periods 0 and t – the intersection universe[1] which includes only matched items. Yet in many product areas old items disappear and new items appear. Constraining the sample to be drawn from this intersection universe is unrealistic. Outlets may sell an item in period 0, but it may not be sold in subsequent periods t.[2] New items may be introduced after period 0 which cannot be compared with a corresponding item in period 0. These items may be variants of the old existing ones, or provide totally new services which cannot be directly compared with anything that previously existed. This universe of all items in periods 0 and t is the dynamic double universe.

21.3 There is a third universe from which prices might be sampled: a replacement universe. The prices of a sample of items in period 0 are determined and their prices are monitored in subsequent periods. If the item is discontinued and there are no longer prices to record for the particular item, prices of a comparable replacement item may be used to continue the series of prices. This universe is a replacement universe that starts with the base period universe, but also includes one-to-one replacements when an item from the sample in the base period is missing in the current period.

21.4 Ideally the replacement item is comparable in quality to the item it replaces. When a comparable replacement is unavailable, a non-comparable one may be selected. Two approaches are possible. An explicit adjustment may be made to the price of either the old or the replacement item for the quality difference, since the two items are of different quality. Alternatively, an assumption may be made that the price change of the ''old'' item, had it continued to exist, would have been the same as that of the overall price change of a targeted group of items from the matched universe. In this second case, an implicit adjustment is being made about the effect on price of the quality change, namely that there is no difference between the price change for the matched group and the quality-adjusted price change of the old item, had it continued to exist.[3] Here, the problem of missing items is considered from the point of view of adjusting prices for quality differences.

21.5 Three practical difficulties emerge. First, there is the problem of explicit quality adjustment between a replacement and old item. The item is no longer consumed, a replacement is found which is not strictly comparable in quality, the differences in quality are identified, and a price has to be put on these differences if the series of prices for the replacement ''new'' item is to be used to continue the series of prices for the ''old'' item.

21.6 Second, in markets where the turnover of items is high, the sample space selected from the matched universe is going to become increasingly unrepresentative of the dynamic universe, as argued in Chapter 8. Even the replacement universe may be inappropriate, as it will comprise series which only replenish themselves when an item needs replacing. Given the rapidly changing technology in many product areas, this universe may be unrepresentative. In such cases, it may be preferable to collect prices, not from matched samples, but from a sample in each period of the main (or a representative sample of) items available, even though they are of a different quality. A comparison between the average prices of such items would be biased if, say, the quality

[1] The terminology is attributable to Dalén (1998a); see also Appendix 8.1.

[2] Its absence may be temporary, being say a seasonal item; ways of treating such temporarily unavailable items are considered in Chapter 8. The concern here, however, is with items that disappear on a permanent basis.

[3] Such methods and their assumption are discussed in detail in Chapter 7.

of the items were improving. The need for, and details of, mechanisms to remove the effects of such changes from the average price comparisons are discussed in Chapter 7.

21.7 Finally, there is the problem of new and disappearing goods and services. These are ones where the new item is not a variant of the old but provides a completely new service. It is not possible to use it as a replacement for an old item by adjusting a price for the quality differential because what it provides is, by definition, something new.

21.8 There are a number of approaches to quality adjustment and these are considered in Chapter 7. One of the approaches is to make explicit adjustments to prices for the quality difference between the old and replacement items using the coefficients or predicted values from hedonic regression equations. Hedonic regressions are regressions of the prices of individual models of a product on their characteristics, for example the prices of television sets on screen size, the possession of stereo or text-retrieval features, etc. The coefficients of such variables provide estimates of the marginal values of different quantifiable characteristics of the product. They can be used to adjust the price of a non-comparable replacement item for differences in its quality as compared with the old item – for example, the replacement television set may have text-retrieval facilities when the previous version did not. It is important that a clear understanding exists of the meaning of such estimated coefficients if they are to be used for quality adjustment, especially given that their use is being promoted.[4] To understand what these estimated parameters mean, it is first necessary to conceive of items as aggregates of their characteristics. Unlike items, characteristics have no separate price attached to them. The price of the item, however, is the price of a "tied" bundle of its characteristics. Then it is necessary to consider what might determine the prices of these characteristics: economic theory points towards examining the demand and supply for characteristics (discussed in paragraphs 21.13 to 21.21) and the interaction of the two to determine an equilibrium price (see paragraphs 21.22 and 21.23). Having developed the analytical framework for such prices, it is then necessary to see what interpretation the economic theory framework allows us to put on these calculated coefficients (see paragraphs 21.24 to 21.28).

21.9 In Chapter 7, paragraphs 7.125 to 7.158, two main approaches were advised for the treatment of product areas with rapid turnover of items. If the sample in period 0 is soon outdated, the matched universe and even replacement universe will become increasingly unrepresentative of the double universe, and repeated sampling from the double universe is required. In this case, either chained indices were advised (see paragraphs 7.153 to 7.158), or one of a number of hedonic indices (see paragraphs 7.132 to 152). Such indices differ from the use of hedonic regression for adjusting prices for quality differences for a missing item. These indices use hedonic regressions, say by including a dummy variable for time on the right-hand side of the equation, to estimate the quality-adjusted price change, as outlined below and in Chapter 7, and build upon the theory in Chapter 17 and paragraphs 21.13 to 21.36. The economic theory of price indices outlined in Chapter 17 is developed to include those tied bundles of good that can be defined in terms of their characteristics as an item in the consumer's utility function. Theoretical consumer price indices are defined which include changes in the prices of characteristics. As with the price indices for goods considered in Chapter 17, there are many formulations that such indices can take, and analogous issues and formulae arise when discussing alternative approaches in paragraphs 21.40 to 21.60.

21.10 The estimation of hedonic regressions and the testing of their statistical properties are facilitated by the availability of user-friendly, yet powerful, statistical and econometric software. There are many standard problems in the estimation of regression equations, which can be resolved using the diagnostic tests available in such software, as discussed by Kennedy (1998) and Maddala (1988). There are, however, matters of functional form, the use of weighted least squares estimators and specification that are specific to the estimation of hedonic equations. Many of these are illustrated in Chapter 7, while Appendix 21.1 to this chapter considers some of the theoretical aspects; see also Gordon (1990), Griliches (1990) and Triplett (1990).

21.11 Finally, in paragraphs 21.61 to 21.68, economic theory is used to advise on the problem of new and disappearing goods and services. This problem arises where differences between existing goods and services and the new goods and services are substantive in nature and cannot be meaningfully compared, even with a quality adjustment. The economic theory of reservation prices is considered and some concern about its practical implementation is expressed.

Hedonic prices and implicit markets

Items as tied bundles of characteristics

21.12 A hedonic regression is a regression equation which relates the prices of items, p, to the quantities of characteristics, given by the vector $z=(z_1, z_2, \ldots, z_n)$, i.e.

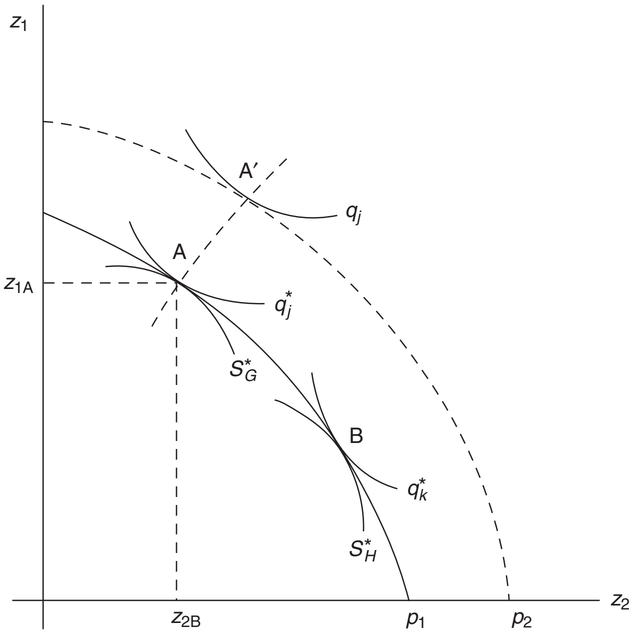
$$p(z)=p(z_1, z_2, \ldots, z_n) \qquad (21.1)$$

where the items are defined in terms of varying amounts of their characteristics. In practice, what will be observed for each item (variant of the product) is its price, its characteristics, and possibly the quantity and thus value sold. Empirical work in this area has been concerned with two issues: estimating how the price of an item changes as a result of unit changes in each characteristic, that is, the estimated coefficients of equation (21.1); and estimating the demand and supply functions for each characteristic. The depiction of an item as a basket of characteristics, each characteristic having it own implicit (shadow) price, requires in turn the specification of a market for such characteristics, since prices result from the workings of markets. Houthakker (1952), Becker

[4] See Boskin, Dullberger, Gordon, Griliches and Jorgenson (1996 and 1998) and Schultze and Mackie (2002).

(1965), Lancaster (1966) and Muth (1966) have all identified the demand for items in terms of their characteristics. The sale of an item is the sale of a tied bundle of characteristics to a consumer whose economic behaviour in choosing between items is depicted as one of choosing between bundles of characteristics.[5] Rosen (1974) further developed the analysis by providing a structural market framework in terms of both producers and consumers. There are two sides: demand and supply. How much of each characteristic is supplied and consumed is determined by the interaction of the demand for characteristics by consumers and the supply of characteristics by producers. These are considered in turn.

Figure 21.1 Consumption and production decisions for combinations of characteristics



The consumer or demand side

21.13 Consider Figure 21.1, from Triplett (1987, p. 634), which shows a simplified characteristics-space between only two characteristics. The hedonic surfaces p_1 and p_2 trace out all the combinations of the two characteristics z_1 and z_2 that can be purchased at prices p_1 and p_2. An indifference curve q_j^* maps the combinations of z_1 and z_2 that the consumer is indifferent about purchasing; that is, the consumer will derive the same utility from any point on the curve. The tangency of q_j^* with p_1 at A is the solution to the utility maximization problem for a given budget (price p_1) and given tastes (reflected in q_j^*).

21.14 The slope of the hedonic surface is the marginal cost to the consumer of acquiring the combination of characteristics and the slope of the utility function is the marginal utility gained from their purchase; the tangency at A is the utility-maximizing combination of characteristics to be purchased at that price. If consumers purchased any other combination of characteristics in the space of Figure 21.1, it would either cost them more to do so or lead to a lower level of utility. Position A′, for example, has more of both z_1 and z_2, and the consumer receives a higher level of utility, being on q_j, but the consumer also has to have a higher budget and pays more, p_2, for being there. Figure 17.1 in Chapter 17 illustrates in goods-space how the consumer would choose between different combinations of outputs, q_1 and q_2. The characteristics-space problem in Figure 21.1 is analogous to the goods-space one, with consumers choosing between combinations of characteristics z_1 and z_2. Note that the hedonic surface depicted in Figure 21.1 is non-linear, so relative characteristic prices are not fixed. The consumer with tastes q_k^* chooses characteristic set B at p_1. Thus the data observed in the market depend on the set of tastes. Triplett (2002) has argued that if tastes were all the same then only one model of, say, a personal computer would be purchased. But in the real world more than one model does exist, reflecting heterogeneous tastes and income levels. Rosen (1974) shows that, of all the characteristic combinations and prices at which they may be offered, the hedonic surface traces out an envelope[6] of tangencies, including q_j^* and q_k^* on p_1 in Figure 21.1. This envelope is simply a description of the locus of the points chosen. Since these points are what economic theory tells us will be observed in the market, assuming rational utility-maximizing consumers, these are the points that will be used to estimate the hedonic regression. Note further that points A and B alone will not allow the regression to determine the price of z_1 relative to z_2 since the observed data will be two combinations of outputs at the same price. The locus of points on an expansion path AA′ would, however, allow this to be determined. Of course, there may be expansion paths for consumers with different tastes, such as B, and this may give rise to conflicting valuations. The overall parameter estimates determined by the regression from transactions observed in the market are an amalgam of such data. Of course, this is just a reflection of the reality of economic life. What arises from this exposition is the fact that the form of the hedonic function is determined in part by the distribution of buyers and their tastes in the market.

21.15 The exposition is now formalized to include parameters for tastes and a numeraire commodity[7] against which combinations of other aggregates are

[5] The range of items is assumed to be continuous in terms of the combinations of characteristics that define them. A non-continuous case can be depicted where the price functions are piece-wise linear and an optimal set of characteristics is obtained by combining the purchases of different items; see Lancaster (1971) and Gorman (1980).

[6] Note that an envelope is more formally defined by letting $f(x, y, k) = 0$ be an implicit function of x and y. The form of the function is assumed to depend on k, the tastes in this case. A different curve corresponds to each value of k in the xy plane. The envelope of this family of curves is itself a curve with the property that it is tangent to each member of the family. The equation of the envelope is obtained by taking the partial derivative of $f(x, y, k)$ with respect to k and eliminating k from the two equations $f(x, y, k) = 0$ and $\partial f(x, y, k)/\partial k = 0$; see Osgood (1925).

[7] The numeraire commodity represents all other goods and services consumed – it represents the "normal" non-hedonic commodities. The price of x is set equal to unity, and $p(z)$ and income are measured using that unit.

selected, following Rosen (1974). The hedonic function $p(z)$ describes variation in the market price of the items in terms of their characteristics. The consumer purchase decision is assumed to be based on utility maximization behaviour, the utility function being given by $U(z, x; \alpha)$ where x is a numeraire commodity, the maximization of utility being subject to a budget constraint given by income y measured as $y = x + p(z)$ (the amount spent on the numeraire commodity and the hedonic products) and α is a vector of the features of individual consumers which describe their tastes. Naturally x in the utility function can be described in terms of $y - p(z)$. Consumers maximize their utility by selecting a combination of quantities of x and characteristics z subject to a budget constraint. The market is assumed to be competitive and consumers are described as "price takers"; they only purchase the one item, so their purchase decision does not influence the market price. The price they pay for a combination of characteristics, vector z, is given by $p(z)$. Since they are optimizing consumers, the combination chosen is such that:

$$[\partial U(z, y - p(z); \alpha)/\partial z_i]/[\partial U(z, y - p(z); \alpha)/\partial x] = \partial p(z)/\partial z_i \equiv p_i(z) \qquad (21.2)$$

where $\partial p(z)/\partial z_i$ is the first derivative of the hedonic function (21.1) with respect to each z characteristic. The coefficients of the hedonic function are equal to their shadow prices, p_i, which equal the utility derived from that characteristic relative to the numeraire good for given budgets and tastes.

21.16 A *value function* θ can be defined as the value of expenditure a consumer with tastes α is willing to pay for alternative values of z at a given utility u and income y, represented by $\theta(z; u, y, \alpha)$. It defines a family of indifference curves relating the z_i to forgone x, "money". For an individual characteristic z_1, θ is the marginal rate of substitution between z_i and money, or the implicit marginal valuation that the consumer with tastes α puts on z_i at a given utility level and income. It is an indication of the reservation demand[8] price for an additional unit of z_i.[9] The price in the market is $p(z)$ and utility is maximized when $\theta(z; u, y, \alpha) = p(z)$, i.e. the purchase takes place where the surface of the indifference curve θ is tangent to the hedonic price surface. If different buyers have different value functions (tastes), some will buy more of a characteristic than others for a given price function, as illustrated in Figure 21.1.

21.17 The joint distribution function of tastes and income sets out a family of value functions each of which, when tangential to the price function, depicts a purchase and simultaneously defines the price function whose envelope is the market hedonic price function. The points of purchase traced out by the hedonic function thus depend on the budget and the tastes of the individual consumer purchasing an individual set of characteristics. If demand functions are to be traced out, the joint probability distribution of consumers with particular budgets and tastes occurring in the market needs to be specified, i.e., $F(y, \alpha)$. This function, along with equation (21.1) allows the demand equations to be represented for each characteristic.

The producer or supply side

21.18 Figure 21.1, reproduced from Triplett (1987), also shows the production side. Consider a revenue-maximizing producer whose revenue maximization problem is given by:[10]

$$R(p, v) \equiv \max_q \left\{ \sum_{n=1}^{N} p_n q_n : (q) \text{ belongs to } S(v) \right\} \qquad (21.3)$$

where $R(p, v)$ is the maximum value of output, $\sum_{n=1}^{N} p_n q_n$, that the establishment can produce, given that it faces the vector of output prices p and given that the vector of inputs v is available for use, using the period t technology. Recall that Figure 17.1 illustrates in goods-space how the producer would choose between different combinations of outputs, q_1 and q_2. In Figure 21.1 the characteristics-space problem is one of producers choosing between combinations of z_1 and z_2 to produce for a particular level of technology and inputs $S(v)$. For a particular producer with level of inputs and technology S_G^* facing a price surface p_1, the optimal production combination is at A. However, a different producer with technology and inputs S_H^* facing a price surface p_1 would produce at B. At these points, the marginal cost of z_1 with respect to z_2 is equal to its marginal price from the hedonic surface, as depicted by the tangency of the point. Production under these circumstances at any other combination would not be optimal. The envelope of tangencies such as S_G^* and S_H^* trace out the production decisions that would be observed in the market from optimizing, price-taking producers and are used as data for estimating the hedonic regressions. The hedonic function can be seen to be determined, in part, by the distribution of technologies of producers, including their output scale.

21.19 Rosen (1974) formalizes the producer side whereby price-taking producers are assumed to have cost functions described by $C(Q, z; \tau)$,[11] where $Q = Q(z)$ is the output scale – the number of units produced by an establishment offering specifications of an item with characteristics z. The producers have to decide which items to produce, i.e., which package of z. To do this, a cost minimization problem is solved which requires τ, equivalent to $S(v)$ above, a vector of the technology of each producer that describes the output combinations each producer can produce with given input costs, using that producer's factors of production and the factor prices. It is the variation in τ across producers that

[8] This is the hypothetical price which just makes the demand for the commodity equal to zero; that is, it is the price which, when inserted into the demand function, sets demand to zero. The utility function is assumed to be strictly concave, so that θ is concave in z and the value function is increasing in z_i at a decreasing rate.

[9] The utility function is assumed strictly concave so that θ is concave in z and the value function is increasing in z_i at a decreasing rate.

[10] The time superscripts are not relevant in this context.

[11] The cost function is assumed to be convex with no indivisibilities, and the marginal cost of producing one more item of a given combination of characteristics is assumed to be positive and increasing. Similarly, the marginal cost of increasing production of each component characteristic is positive and non-decreasing.

distinguishes producer A's decision as to which combination of z to produce from that of producer B in Figure 21.1. Producers are optimizers who seek to maximize profits, given by:

$$Qp(z) - C(Q, z; \tau) \quad (21.4)$$

by selecting Q and z optimally. The supplying market is assumed to be competitive, and producers are price takers so the producer cannot influence price by a production decision. The producer's decision as to how much to produce of each z is determined by the price of z, assuming that the producer can vary Q and z in the short run.[12] Dividing the expression (21.4) by Q and setting it equal to zero, the first-order profit-maximizing conditions are given by:

$$\frac{\partial p}{\partial z_i} = p_i = \frac{C_{zi}(Q, z; \tau)}{Q} \quad (21.5)$$

where $p = p(z_1, z_2, \ldots, z_n)$ from equation (21.1).

21.20 The marginal *unit revenue* from producing characteristic z_i is given by its shadow price in the price function and its marginal cost of production. In the producer case, the probability distribution of the technologies of firms $G(\tau)$ is necessary if the overall quantity supplied of items with given sets of characteristics is to be revealed. Since it is a profit maximization problem to select the optimal combination of characteristics to produce, marginal revenue from the additional attributes must equal their marginal cost of production per unit sold. Quantities are produced up to the point where unit revenues $p(z)$ equal their marginal production cost, evaluated at the optimum bundle of characteristics supplied.

21.21 While a value function was considered for consumers, producers require an *offer function* $\phi(z; \pi, \tau)$. The offer price is the price the seller is willing to accept for various values of z at constant profit level π, when quantities produced are optimally chosen, while $p(z)$ is the maximum price obtainable from those models in the market. Producer equilibrium is characterized by a tangency between a profit characteristics indifference surface and the market characteristics price surface where $p_i(z_i) = \varphi_{zi}(z; \pi z; \pi)$ and $p_i(z_i) = \varphi_{zi}(z; \pi z; \pi)$. Since there is a distribution of technologies $G(\tau)$, the producer equilibrium is characterized by a family of offer functions that envelop the market hedonic price function. The varying τ will depend on different factor prices for items produced in different countries, multi-product firms with economies of scale and differences in the technology, be it the quality of capital, labour or intermediate inputs and their organization. A family of production surfaces is defined for different values of τ.

[12] Rosen (1974) considered two other supply characterizations: the short run in which only M is variable and the long run in which plants can be added and retired. The determination of equilibrium supply and demand is not straightforward. A function $p(z)$ is required such that market demand for all z will equate to market supply and clear the market. But demand and supply depend on the whole $p(z)$, since any adjustment to prices to equate demand and supply for one combination of items will induce substitutions and changes for others. Rosen (1974, pp. 44–48) discusses this in some detail.

Equilibrium

21.22 The theoretical framework first defines each item as a point on a plane of several dimensions made up by the $z_1, z_2, \ldots, z_n$ quality characteristics; each item is a combination of values $z_1, z_2, \ldots, z_n$. If only two characteristics define the item, then each point in the positive space of Figure 21.1 would define an item. The characteristics are not bought individually, but as bundles of characteristics tied together to make up an item. It is assumed that the markets are differentiated so that there is a wide range of choices to be made.[13] The market is also assumed to be perfectly competitive, with consumers and producers as price takers undertaking optimizing behaviour to decide which items (tied sets of characteristics) to buy and sell. Competitive markets and optimizing behaviour are assumed so that the quantity demanded of an item equals the quantity supplied. Since the items are made up of characteristics z, the quantity demanded of all characteristics z must equal their quantity supplied. It has been shown that consumers' and producers' choices or "locations" on the plane will be dictated by consumer tastes and producer technology. Tauchen and Witte (2001, p. 4) show that the hedonic price function will differ across markets in accordance with the means and variances (and in some cases also higher moments) of the distributions of household and firm characteristics.

21.23 Rosen (1974, p. 44) notes that a buyer and seller are perfectly matched when their respective value and offer functions are tangential to each other. The common gradient at that point is given by the gradient of the market clearing implicit price function $p(z)$. The consumption and production decisions are seen in the value and offer functions to be jointly determined, for given $p(z)$, by $F(y, \alpha)$ and $G(\tau)$. In competitive markets there is a simultaneity in the determination of the hedonic equation, as the distribution of $F(y, \alpha)$ and $G(\tau)$ helps determine the quantities demanded and supplied, as well as the slope of the function. Although the decisions made by consumers and producers are as price takers, the prices taken are those from the hedonic function. There is a sense in which the hedonic function and its shadow prices emerge from the operations of the market. The product markets implicitly reveal the hedonic function. Since consumers and producers are optimizers in competitive markets, the hedonic function, in principle, gives the minimum price of any bundle of characteristics. Given all of this, Rosen (1974, p. 44) asked: What do hedonic prices mean?

What hedonic prices mean

21.24 It would be convenient if, for CPI construction, the estimated coefficients from hedonic regressions were estimates of the marginal utility based on a characteristic or user value. But theory tells us that this is not the case and that the interpretation is not clear.

21.25 There used to be an erroneous perception, in the 1960s, that the coefficients from hedonic methods

[13] So that choices among combinations of z are continuous, assume further that z possesses continuous second-order derivatives.

represented user values as opposed to resource costs. Rosen (1974), as has been shown, found that hedonic coefficients generally reflect both user values and resource costs, both supply and demand situations. The ratios of these coefficients may reflect consumers' marginal rates of substitution or producers' marginal rates of substitution (transformation) for characteristics. There is what is referred to in econometrics as an "identification" problem in which the observed prices and quantities are jointly determined by supply and demand considerations, and their underlying sources cannot be separated. The data collected on prices jointly arise from variations in demand by different consumers with different tastes and preferences, and from variations in supply by producers with different technologies.

21.26 First, it is necessary to come to terms with this simultaneity problem. Hedonic regressions are an increasingly important analytical tool, one implicitly promoted by the attention given to it in this manual, but also promoted in manuals by organizations such as the OECD (see Triplett (2002)), Eurostat (2001a) and widely used by the United States Bureau of Labor Statistics; see Kokoski, Waehrer and Rozaklis (2001) and Moulton (2001). So how do economists writing on the subject shrug their intellectual shoulders in the light of these findings?

Rosen (1974, p. 43) refers to the hedonic function as:

> ... a joint envelope of a family of value functions and another family of offer functions. An envelope function by itself reveals nothing about the underlying members that generate it; and they in turn constitute the generating structure of the observations.

Griliches (1988, p. 120) notes the following:

> My own view is that what the hedonic approach tries to do is to estimate aspects of the budget constraint facing consumers, allowing thereby the estimation of "missing" prices when quality changes. It is not in the business of estimating utility functions per se, though it can also be useful for these purposes ... what is being estimated is the actual locus of intersection of the demand curves of different consumers with varying tastes and the supply curves of different producers with possible varying technologies of production. One is unlikely, therefore, to be able to recover the underlying utility and cost functions from such data alone, except in very special circumstances.

Triplett (1987) states:

> It is well-established – but still not widely understood – that the form of $h(\cdot)$ [the hedonic function] cannot be derived from the form of $Q(\cdot)$ and $t(\cdot)$ [utility and production functions], nor does $h(\cdot)$ represent a "reduced form" of supply and demand functions derived from $Q(\cdot)$ and $t(\cdot)$.

Diewert (2003a, p. 320), with his focus on the consumer side, says:

> Thus I am following Muellbauer's (1974, p. 977) example where he says that his "approach is unashamedly one-sided; only the demand side is treated ...". Its subject matter is therefore rather different from that of the recent paper by Sherwin Rosen. The supply side and simultaneity problems which may arise are ignored.

21.27 Diewert's (2003a) approach is of interest since the derivation of estimates of hedonic coefficients is one that follows the demand side only. It is a useful exercise to consider the conditions under which the hedonic coefficients are determined by only demand-side or supply-side factors – the circumstances under which clear explanations would be valid. The problem is that because the coefficients of a hedonic function are the outcome of the interaction of consumer and producer optimizing conditions, it is not possible to interpret the function only in terms of, say, producer marginal costs or consumer marginal values. Suppose, however, that the *production technology* τ is the same for each producing establishment. Buyers differ but sellers are identical. Then instead of there being a confusing family of offer functions, there is a unique offer function with the hedonic function describing the prices of characteristics the firm will supply with the given ruling technology to the current mixture of tastes. The function $p(z)$ becomes the offer function since there is no distribution of τ to confuse it. There are different tastes on the consumer side, so what appears in the market is the result of firms trying to satisfy consumer preferences, all for a constant technology and profit level; the structure of supply is revealed by the hedonic price function. In Figure 21.1 only the expansion path traced out by, say, S_H^* akin to AA′, would be revealed. Now suppose that sellers differ, but that buyers' tastes α are identical. Here the family of value functions collapses to be revealed as the hedonic function $p(z)$ which identifies the structure of demand, such as AA′ in Figure 21.1.[14] Triplett (1987, p. 632) notes that of these possibilities, uniformity of technologies is the most likely, especially when access to technology is unrestricted in the long run, while uniformity of tastes is unlikely. There may, of course, be segmented markets where tastes are more uniform, to which specific sets of items are tailored and for which hedonic equations can be estimated for individual segments.[15]

21.28 The analysis in paragraphs 21.12 to 21.27 demonstrates the ambiguity surrounding the interpretation of hedonic coefficients. This analysis is needed since issues relating to the estimation of the underlying supply and demand functions for characteristics have implications for the estimation of hedonic functions. Appendix 21.1 considers identification and estimation issues in this light. The next section provides an alternative theoretical derivation, based on Diewert (2003a), which shows the assumptions required for a demand-based (consumer-based) interpretation.

[14] Correspondingly, if the supply curves were perfectly inelastic, so that a change in price would not affect the supply of any of the differentiated products, then the variation in prices underlying the data and feeding the hedonic estimates would be determined by demand factors. The coefficients would provide estimates of user values. Similarly, if the supplying market were perfectly competitive, the estimates would be of resource costs. None of the price differences between differentiated items would be attributable to, say, novel configurations of characteristics, and no temporary monopoly profit would be achieved as a reward for this, or as a result of the exercise of market power; see Berndt (1991).

[15] Berry, Levinsohn and Pakes (1995) provide a detailed and interesting example for cars, in which makes are used as market segments. Tauchen and Witte (2001) provide a systematic theoretical study of estimation issues for supply, demand and hedonic functions, where consumers and producers and their transactions are indexed across communities.

An alternative, consumer-based hedonic theoretical formulation

21.29 This section takes a consumer-based approach to deriving theoretical hedonic functions. It assumes:

- that every consumer has the *same separable sub-utility function*, $f(z_1, \ldots, z_N)$ that gives the consumer the sub-utility $Z = f(z)$ from the purchase of one unit of the complex hedonic commodity that has the vector of characteristics $z \equiv (z_1, \ldots, z_N)$;[16]
- the sub-utility that the consumer gets from consuming Z units of the hedonic commodity is combined with the consumption of X units of a composite "other" commodity to give the consumer an overall utility of $u = U^t(X, Z)$ in period t, where U^t is the period t "macro" utility function. Rosen (1974, p. 38) normalized the price of X to be unity. This is not required in the present approach. Instead, there is an explicit period t price, p^t, for one unit of the general consumption commodity X.

21.30 The approach starts by considering the set of X and Z combinations that can yield the consumer's period t utility level, u^t. This is the set $\{(X, Z)\colon U^t(X, Z) = u^t\}$, which is the consumer's period t indifference curve over equivalent combinations of the general consumption commodity X and the hedonic commodity Z. The equation $U^t(X, Z) = u^t$ for X is solved as a function of u^t and Z; i.e.[17]

$$X = g^t(u^t, Z) \qquad (21.6)$$

It is assumed that the indifference curve slopes downward, and the stronger assumption is made that g^t is differentiable with respect to Z and

$$\partial g^t(u^t, Z)/\partial Z < 0 \qquad (21.7)$$

Let p^t and P^t be the prices for one unit of X and Z, respectively, in period t. The *consumer's period* t *expenditure minimization problem* may be defined as follows:

$$\begin{aligned} &\min_{X,Z} \{ p^t X + P^t Z : X = g^t(u^t, Z)\} \\ &= \min_Z \{ p^t g^t(u^t, Z) + P^t Z\} \end{aligned} \qquad (21.8)$$

The first-order necessary condition for Z to solve equation (21.8) is:

$$p^t \partial g^t(u^t, Z)/\partial Z + P^t = 0 \qquad (21.9)$$

Equation (21.9) can be rearranged to give the price of the hedonic aggregate P^t as a function of the period t utility level u^t and the price of general consumption p^t:

$$P^t = -p^t \partial g^t(u^t, Z)/\partial Z > 0 \qquad (21.10)$$

where the inequality follows from assumption (21.7). The right-hand side of the equation (21.10) can now be interpreted as the consumer's *period* t *willingness to pay price function:*

$$w^t(Z, u^t, p^t) \equiv -p^t \partial g^t(u^t, Z)/\partial Z \qquad (21.11)$$

21.31 Thus, for each point (indexed by Z) on the consumer's period t indifference curve, equation (21.11) gives the amount of money the consumer would be willing to pay per unit of Z in order to stay on the same indifference curve, which is indexed by the utility level u^t. The *period* t *willingness to pay value function* v^t can now be defined as the product of the quantity of Z consumed times the corresponding per unit willingness to pay price, $w^t(Z, u^t, p^t)$:

$$v^t(Z, u^t, p^t) \equiv Z w^t(Z, u^t, p^t) = -Z p^t \partial g^t(u^t, Z)/\partial Z \qquad (21.12)$$

where the last equality follows using equation (21.11). The function v^t is the counterpart to Rosen's (1974, p. 38) value or bid function; it gives us the amount of money the consumer is willing to pay in order to consume Z units. All the above algebra has an interpretation that is independent of the hedonic model; it is simply an exposition of how to derive a willingness to pay price and value function using a consumer's preferences defined over two commodities.

21.32 It is assumed now that the consumer has a separable sub-utility function, $f(z_1, \ldots, z_N)$, which gives the consumer the sub-utility $Z = f(z)$ from the purchase of one unit of the complex hedonic commodity[18] that has the vector of characteristics $z \equiv (z_1, \ldots, z_N)$. Note that it has been assumed that the function f is time invariant. Let the consumer's period t utility function be $U^t(X, f(z))$. The above algebra on willingness to pay is still valid. In particular, the new period t willingness to pay price function, for a particular model with characteristics $z = (z_1, \ldots, z_N)$, is:

$$w^t(f(z), u^t, p^t) \equiv -p^t \partial g^t(u^t, f(z))/\partial Z \qquad (21.13)$$

The new period t willingness to pay value function (which is the amount of money the consumer is willing

[16] It is not assumed that all possible models exist in the marketplace. In fact, we will assume that only a finite set of models exist in each period. It is assumed, however, that the consumer has preferences over all possible models, where each model is indexed by its vector of characteristics, $z = (z_1, \ldots, z_N)$. Thus each consumer will prefer a potential model with characteristics vector $z^1 = (z_1^1, \ldots, z_N^1)$ over another potential model with the characteristics vector $z^2 = (z_1^2, \ldots, z_N^2)$ if and only if $f(z^1) > f(z^2)$.

[17] If the period t indifference curve intersects both axes, then $g^t(u^t, Z)$ will only be defined for a range of non-negative Z up to an upper bound.

[18] If a consumer purchases, say, two units of a model at price P that has characteristics $z_1, \ldots, z_N$, then we can model this situation by introducing an artificial model that sells at price $2P$ and has characteristics $2z_1, \ldots, 2z_N$. Thus the hedonic surface, $Z = f(z)$, consists of only the most efficient models including the artificial models. We do not assume that $f(z)$ is a quasi-concave or concave function of z. In normal consumer demand theory, $f(z)$ can be assumed to be quasi-concave without loss of generality because linear budget constraints and the assumption of perfect divisibility will imply that "effective" indifference curves enclose convex sets. As Rosen (1974, pp. 37–38) points out, however, in the case of hedonic commodities, the various characteristics cannot be untied. Moreover, perfect divisibility cannot be assumed and not all possible combinations of characteristics will be available in the marketplace. Thus the usual assumptions made in "normal" consumer demand theory are not satisfied in the hedonic context. Note also that while we place a smoothness assumption on the macro functions $g^t(u, Z)$, namely the existence of the partial derivative $\partial g^t(u, Z)/\partial Z$), we do not place any smoothness restrictions on the hedonic sub-utility function $f(z)$.

to pay to have the services of a model with characteristics vector z) is:

$$v^t(f(z), u^t, p^t) \equiv f(z)w^t(f(z), u^t, p^t) = -f(z)p^t \partial g^t(u^t, f(z))/\partial Z \qquad (21.14)$$

21.33 Now suppose that there are K^t models available to the consumer in period t, where model k sells at the per unit price of P_k^t and has the vector of characteristics $z_k^t \equiv (z_{1k}^t, \ldots, z_{Nk}^t)$ for $k = 1, 2, \ldots, K$. If the consumer purchases a unit of model k in period t, then the model price P_k^t can be equated to the appropriate willingness to pay value defined by equation (21.14), where z is replaced by z_k^t; i.e., the following equation should hold:

$$P_k^t = -f(z_k^t)p^t \partial g^t(u^t, f(z_k^t))/\partial Z \quad \text{for } t = 1, \ldots, T; k = 1, \ldots, K^t \qquad (21.15)$$

What is the meaning of the separability assumption? Suppose the hedonic commodity is a car and suppose that there are only three characteristics: number of seats in the vehicle, fuel economy and horsepower. The separability assumption means that the consumer can trade off these three characteristics and determine the utility of any car with any mix of these three characteristics, independently of his or her other choices of commodities. In particular, the utility ranking of car models is independent of the number of children the consumer might have or what the price of petrol might be. Obviously, the separability assumption is not likely to be exactly satisfied in the real world, but this somewhat restrictive assumption is required to make our model tractable.

21.34 Another aspect of our model needs some further explanation. It is being explicitly assumed that consumers cannot purchase fractional units of each model.; they can purchase only a non-negative integer amount of each model. That is, indivisibilities are being explicitly assumed on the supply side of our model. Thus, in each period, there is only a finite number of models of the hedonic commodity available. While the consumer is assumed to have continuous preferences over all possible combinations of characteristics $(z_1, \ldots, z_N)$ in each period, there is only a finite number of isolated models available on the market.

21.35 At this point, the model is further specialized. It is assumed that every consumer has the same hedonic sub-utility function[19] $f(z)$ and consumer i has the following *linear indifference curve macro utility function* in period t:

$$g_i^t(u_i^t, Z) \equiv -a^t Z + b_i^t u_i^t \quad \text{for } t = 1, \ldots, T \text{ and } i = 1, \ldots, I \qquad (21.16)$$

where a^t and b_i^t are positive constants.

For each period t and each consumer i, the period t indifference curve between combinations of X and Z is linear, with the constant slope $-a^t$ being the same for all consumers.[20] Note that this slope is allowed to change over time. Now differentiate equation (21.16) with respect to Z and substitute this partial derivative into equation (21.15). The resulting equation is:[21]

$$P_k^t = p^t a^t f(z_k^t) \quad \text{for } t = 1, \ldots, T \text{ and } k = 1, \ldots, K^t \qquad (21.17)$$

Define the aggregate price of one unit of Z in period t as:[22]

$$r_t \equiv p^t a^t \quad \text{for} \quad t = 1, \ldots, T \qquad (21.18)$$

Now substitute equation (21.18) into equation (21.17) in order to obtain our *basic system of hedonic equations*.[23]

$$P_k^t = r_t f(z_k^t) \quad \text{for } t = 1, \ldots, T \text{ and } k = 1, \ldots, K^t \qquad (21.19)$$

21.36 All that is needed is to postulate a functional form for the hedonic sub-utility function f and add a stochastic specification to equation (21.19) to yield a basic hedonic regression model. The unknown parameters in f along with the period t hedonic price parameters r_t can then be estimated.[24] It is possible to generalize the above model, but get the same model

[19] The sameness assumption is very strong and needs some justification. This assumption is entirely analogous with the assumption that consumers have the same homothetic preferences over, say, food. Although this assumption is not justified for some purposes, it suffices for the purpose of constructing a price index for food, since we are mostly interested in capturing the substitution effects in the aggregate price of food as the relative prices of food components vary. In a similar fashion, we are interested in determining how the "average" consumer values a faster computer speed against more memory; i.e., we are primarily interested in hedonic substitution effects.

[20] We do not require a linear indifference curve globally, but only locally over a certain range of purchases. Alternatively, we can view the linear indifference curve as providing a first-order approximation to a non-linear indifference curve.

[21] Comparing equation (21.17) with equation (21.15), it can be seen that the simplifying assumptions (21.16) enable us to get rid of the terms $\partial g^t(u_i^t, f(z_k^t))/\partial Z$, which depend on individual consumer indifference curves between the hedonic commodity and other commodities. If we had individual household data on the consumption of hedonic and other commodities, then we could use normal consumer demand techniques in order to estimate the parameters that characterized these indifference curves.

[22] There has been a switch to subscripts from superscripts in keeping with the conventions for parameters in regression models; i.e., the constants r_t will be regression parameters in what follows. Note also that r_t is the product of the price of the "other" commodity p^t times the period t slope parameter a^t. We need to allow this slope parameter to change over time in order to be able to model the demand for high-technology hedonic commodities, which have been falling in price relative to "other" commodities; i.e., we think of a^t as decreasing over time for high-technology commodities.

[23] The basic model ends up being very similar to one of Muellbauer's (1974, pp. 988–989) hedonic models; see in particular his equation (32).

[24] It is possible to rework the above theory and give it a producer theory interpretation. The counterpart to the expenditure minimization problem (21.8) is now the following profit maximization problem: $\max_{x,z} \{P^t Z - w^t X : X = g^t(k^t, Z)\}$ where Z is hedonic output and P^t is a period t price for one unit of the hedonic output, w^t is the period t price of a variable input and X is the quantity used of it, k^t is the period t quantity of a fixed factor (capital, say) and g^t is the firm's factor requirements function. Assuming that $Z = f(z)$, we end up with the following producer theory counterpart to equation (21.15): $P_k^t = f(z_k^t)\,\partial g^t(k^t, f(z_k^t))/\partial Z$. The counterpart to assumption (21.16) is, for firm i, $g_i^t(k_i^t, Z) \equiv a^t Z - b_i^t k_i^t$ and the counterpart to equation (21.17) becomes $P_k^t = w^t a^t f(z_k^t)$. The producer theory model assumptions are, however, not as plausible as the corresponding consumer theory model assumptions. In particular, it is not very likely that each producer will have the same period t aggregate price for a unit of variable input w^t and it is not very likely that each firm producing in the hedonic market will have the same technology parameter a^t. The key assumption that will not generally be

(21.19) if the composite "other" commodity X is replaced by $h(x)$, where x is a consumption vector and h is a linearly homogeneous, increasing and concave aggregator function. Instead of equation (21.17), under these new assumptions, the following equation results:

$$P_k^t = c(p^t)a^t f(z_k^t) \quad \text{for } t=1,\ldots,T \quad \text{and } k=1,\ldots,K^t \tag{21.20}$$

where p^t is now the vector of prices for the x commodities in period t and c is the unit cost or expenditure function that is dual to h.[25] Now redefine r_t as $c(p^t)a^t$ and the basic system of hedonic equations (21.19) is still obtained. Equation (21.19) has one property that is likely to be present in more complex and realistic models of consumer choice. This property is that the model prices in period t are homogeneous of degree one in the general price level p^t. Thus if p^t is replaced by λp^t for any $\lambda > 0$ (think of a sudden hyperinflation where λ is large), then equations (21.17) and (21.19) imply that the model prices should become λP_k^t. Note that this homogeneity property will not hold for the following additive hedonic model:

$$P_k^t = r_t + f(z_k^t) \quad \text{for } t=1,\ldots,T \text{ and } k=1,\ldots,K^t \tag{21.21}$$

Thus hedonic regressions based on the linear model (21.21) may be ruled out on a priori grounds. Note that hedonic models that take the logarithm of the model price P_k^t as the dependent variable will tend to be consistent with basic hedonic equations (21.19), whereas linear models like (21.21) will not be consistent with the normal linear homogeneity properties implied by microeconomic theory.

Hedonic indices

21.37 It was noted above that hedonic functions are required for two purposes with regard to a quality adjustment. The first is when an item is no longer available and the replacement item, of which the price is used to continue the series, is of a different quality to the original price basis. The differences in quality can be established in terms of different values of a subset of the z price-determining variables. The coefficients from the hedonic regressions, as estimates of the monetary value of additional units of each quality component z, can then be used to adjust the price of, say, the old item so that it is comparable with the price of the new item[26] – so that, again, like is compared with like. This process could be described as "patching", in that an adjustment is needed to the price of the old (or new replacement) series for the quality differences, to enable the new series to be patched onto the old. A second use of hedonic functions is for estimating *hedonic indices*. These are suitable when the pace and scale of replacements of items are substantial and when an extensive use of patching might lead to extensive errors if there were some error or bias in the quality adjustment process and to sampling from a biased replacement universe (as outlined in paragraphs 21.12 to 21.36). Hedonic indices use data in each period from a refreshed sample of items, which should include those with a substantial share of sales expenditure – with sampling in each period from the double universe. There is no need to establish a preselected set of items for matching and for price collectors to keep collecting prices for that set. What is required are samples of items to be redrawn in each month along with information on their prices, characteristics and, possibly, quantities/values. The "partialling out" in the hedonic regressions controls for quality differences, as opposed to the matching of price by the price collectors. There are a number of procedures for estimating hedonic indices and these are briefly considered below.

Theoretical characteristics price indices

21.38 Theoretical cost of living indices are defined in Chapter 17 and practical index number formulae are considered as estimates of these indices. Theoretical cost of living index numbers are defined here not just on the goods produced, but also on their characteristics. The Konüs (1924) family of *true cost of living indices* pertaining to two periods, where the consumer faces the strictly positive price vectors $p^0 \equiv (p_1^0,\ldots,p_N^0)$ and $p^1 \equiv (p_1^1,\ldots,p_N^1)$ in periods 0 and 1 respectively, was defined in Chapter 17 as the ratio of the minimum costs of achieving the same utility level $u \equiv f(q)$, where $q \equiv (q_1,\ldots,q_N)$ is a positive reference quantity vector; i.e.,

$$P_K(p^0,p^1,q) \equiv C[u,p^1]/C[u,p^0] = C[f(q),p^1]/C[f(q),p^0] \tag{21.22}$$

For theoretical indices in characteristic space, the revenue functions are also defined over goods made up of bundles of characteristics represented by the hedonic function:[27]

$$\begin{aligned} P_K(p^0,p^1,q) &\equiv C[u,p^1,p(z_1)]/C[u,p^0,p(z_0)] \\ &= C[f(q),p^1,p(z_1)]/C[f(q),p^0,p(z_0)] \end{aligned} \tag{21.23}$$

21.39 The theoretical price index defined by equation (21.23) is a ratio of the period 1 to period 0 hypothetical costs to consumers of achieving a given utility. Equation

satisfied in the producer context is that each producer is able to produce the entire array of hedonic models; whereas, in the consumer context, it is quite plausible that each consumer has the possibility of purchasing and consuming each model.

[25] Define c as $c(p^t) \equiv \min_x \{p^t x : h(x) = 1\}$, where $p^t x$ denotes the inner product between the vectors p^t and x.

[26] Various mechanisms for such adjustments are outlined in paragraphs 7.103 to 7.109, and Triplett (2002). They include using the coefficients from the salient set of characteristics or using the predicted values from the regression as a whole and, in either case, making the adjustment to the old for comparison with the new, or to the new for comparison with the old, or some effective average of the two.

[27] Triplett (1987) and Diewert (2002d), following Pollak (1975), consider a two-stage budgeting process whereby that portion of utility concerned with items defined as characteristics has its theoretical index defined in terms of a cost-minimizing selection of characteristics, conditioned on an optimum output level for composite and hedonic commodities. These quantities are then fed back into the second-stage maximization of overall revenue.

(21.23) incorporates substitution effects: if the prices of some characteristics increase more than others, then utility-maximizing consumers can switch their output mix of characteristics in favour of such characteristics. The numerator in equation (21.23) is the cost of the maximum utility that the consumer could attain if faced with the commodity prices and implicit hedonic shadow prices of period 1, p^1 and $p(z^1)$, while the denominator in equation (21.23) is the maximum utility that the consumer could attain if faced with the commodity and characteristic prices of period 0, p^0 and $p(z^0)$. Note that all the variables in the numerator and denominator functions are exactly the same, except that the commodity price and characteristic price vectors differ. This is a defining characteristic of a price index. As with the economic indices in Chapter 15, there is of course an entire family of indices depending on which reference utility level is chosen. Some explicit formulations are considered in paragraphs 21.48 to 21.58, including a base period 0 reference level and a current period 1 reference level analogous to the derivation of the Laspeyres and Paasche indices in Chapter 17. Before considering such hedonic indices, two simpler formulations are first considered: hedonic regressions using dummy variables on time (paragraphs 21.40 to 21.42), and hedonic imputation indices (paragraphs 21.43 to 21.47). They are simple, and widely used because they require no information on quantities or weights. They also do not require matched data, so can be used when resampling all the data. Yet their interpretation from economic theory is therefore more limited on account of this. As will be shown in Appendix 21.1, however, weighted formulations are possible using a weighted least squares estimator.

Hedonic regressions and dummy variables of time

21.40 Let there be K characteristics of a product and let model or item i of the product in period t have the vector of characteristics $z_i^t \equiv [z_{i1}^t, \ldots, z_{iK}^t]$ for $i=1,\ldots,I$ and $t=1,\ldots,T$. Denote the price of model i in period t by p_i^t. A hedonic regression of the price of model i in period t on its characteristics set z_i^t is given by:

$$\ln p_i^t = \gamma_0 + \sum_{t=2}^{T} \gamma^t D^t + \sum_{k=1}^{K} \beta_k z_{ik}^t + \varepsilon_i^t \qquad (21.24)$$

where D^t are dummy variables for the time periods, D^2 being 1 in period $t=2$, zero otherwise; D^3 being 1 in period $t=3$, zero otherwise, etc. The coefficients γ^t are estimates of quality-adjusted price changes, having controlled for the effects of variation in quality (via $\sum_{k=1}^{K}\gamma_k z^t ki$) – but see Goldberger (1968) and Teekens and Koerts (1972) for the adjustment for estimation bias.

21.41 The above approach uses the dummy variables on time to compare prices in period 1 with prices in each subsequent period. In doing so, the β_k parameters are constrained to be constant over the period $t=1,\ldots,T$. Such an approach is fine retrospectively, but in real time the index may be estimated as a fixed base or chained base formulation. The *fixed base* formulation would estimate the index for period 1 and 2, $I_{1,2}$, using equation (21.24) for $t=1$, 2; the index for period 3, $I_{1,3}$, would use equation (21.24) for $t=1$, 3; the index for period 4, $I_{1,4}$, using equation (21.24) for $t=1$, 4 and so forth. The dummy variable in such cases would take values of 1 for the current period and 0 for the price reference period. For example, for $I_{1,4}$ it would be 1 for observations in period 4 and 0 otherwise, i.e. for observations in period 1. Only data in periods 1 and 4 are used for the estimated equation. The coefficient of the dummy variable is an estimate of quality-adjusted price change. In each case, the index constrains the estimated coefficients on the quality characteristics to be the same for the current and price reference periods, periods 1 and 4. A fixed base bilateral comparison using equation (21.24) makes use of the constrained parameter estimates over the two periods of the price comparison. *A chained* formulation would estimate $I_{1,4}$, for example, as the product of a series of links: $I_{1,4} = I_{1,2} \times I_{2,3} \times I_{3,4}$. Each successive binary comparison, or link, is combined by successive multiplication. The index for each link is estimated using equation (21.24). Because the periods of time being compared are close, it is generally more likely that the constraining of parameters required by chained time dummy hedonic indexes are considered to be less severe than that required of their fixed base counterparts.

21.42 There is no explicit weighting in these formulations, and this is a serious disadvantage. In practice, cut-off sampling might be employed to include only the most important items. If sales data are available, a weighted (by relative sales shares; see Appendix 21.1) least squares estimator, as opposed to an ordinary least squares estimator (OLS), should be used.[28] Matched data are not required for this method; items can be resampled in each period to include new technologies.

Hedonic imputation indices

21.43 An alternative approach for a comparison between period 1 and t is to estimate a hedonic regression for period t, and insert the values of the characteristics of each model existing in period 1 into the period t regression to predict, for each item, its price $\hat{p}_i^t(z_i^1)$. This would generate predictions of the price of items existing in period 1, at period t shadow prices, $\hat{p}_i^t(z_i^1), i=1,\ldots,N$. These prices (or an average) can be compared with (the average of) the actual prices of models $i=1,\ldots,N$ models in period 1. The averages may be arithmetic, as in a Dutot index, or geometric, as in a Jevons index. The arithmetic formulation is defined as follows:

$$\frac{\sum_{i=1}^{N} (1/N)\hat{p}_i^t(z_i^1)}{\sum_{i=1}^{N} (1/N)p_i^1(z_i^1)} \qquad (21.25a)$$

21.44 Alternatively, the characteristics of models existing in period 1 can be inserted into a regression for period t. Predicted prices of period t items generated at

[28] Ioannidis and Silver (1999) and Bode and van Dalén (2001) compared the results from these different estimators, finding notable differences, but not in all cases; see also Silver and Heravi (2003).

period 1 shadow prices (or an average) can be compared with (the average of) the actual prices in period t:

$$\frac{\sum_{i=1}^{N}(1/N)p_i^t(z_i^t)}{\sum_{i=1}^{N}(1/N)\hat{p}_i^1(z_i^t)} \tag{21.25b}$$

21.45 For a fixed base bilateral comparison using either formula (21.25a) or (21.25b), the hedonic equation need only be estimated for one period. The denominator in formula (21.25a) is the average observed price in period 1, which should be equal to the average price that a hedonic regression based on period 1 data will predict using period 1 characteristics. The numerator, however, requires an estimated hedonic regression to predict period 1 characteristics at period t hedonic prices. Similarly, in formula (21.25b) a hedonic regression is required only for the denominator. For reasons analogous to those explained in Chapter 15, a symmetric average of these indices should have some theoretical support.

21.46 Note that all the indices described in paragraphs 21.40 to 21.45 can use matched data or all the data available in each period. If there is a new item in, say, period 4, it is included in the data set and its quality differences controlled for by the regression. Similarly, if old items drop out, they are still included in the indices in the periods in which they exist. This is part of the natural estimation procedure and differs from the use of hedonic regressions only for price adjustments to non-comparable items.

21.47 As with the dummy variable approach, there is no need for matched data. Yet there is also no explicit use of quantity weighting in these formulations, and this is a serious disadvantage. Were data on quantities or values available, it is immediately apparent that such weights could be attached to the individual $i=1,\ldots,N$ prices or their estimates. This is considered in the next section.

Superlative and exact hedonic indices

21.48 In Chapter 17, Laspeyres and Paasche bounds were defined on a theoretical basis, as were superlative indices, which treat both periods symmetrically. These superlative formulae included the Fisher index, which was seen in Chapter 16 to have desirable axiomatic properties. Furthermore, the Fisher index was supported from economic theory as a symmetric average of the Laspeyres and Paasche bounds, and was found to be the most suitable such average of the two on axiomatic grounds. The Törnqvist index also possessed desirable axiomatic properties, seemed to be best from the stochastic viewpoint, and also did not require strong assumptions for its derivation from the economic approach as a superlative index. The Laspeyres and Paasche indices were found to correspond to (be exact for) underlying (Leontief) aggregator functions with no substitution possibilities, while superlative indices were exact for flexible functional forms, including the quadratic and translog forms for the Fisher and Törnqvist indices respectively. If data on prices, characteristics and quantities are available, then analogous approaches and findings arise for hedonic indices; see Fixler and Zieschang (1992) and Feenstra (1995). Exact bounds on such an index were defined by Feenstra (1995). Consider the theoretical index in equation (21.23), but now only defined over items in terms of their characteristics. The prices are still of items, but they are wholly defined through $p(z)$. An arithmetic aggregation for a linear hedonic equation finds that a Laspeyres upper bound (as quantities supplied *decrease* with increasing relative prices) is given by:

$$\frac{\sum_{i=1}^{N}x_i^{t-1}\hat{p}_i^t}{\sum_{i=1}^{N}x_i^{t-1}p_i^{t-1}}=\sum_{i=1}^{N}s_i^{t-1}\left(\frac{\hat{p}_i^t}{p_i^{t-1}}\right)\geq\frac{C(u^{t-1},p(z)^t)}{C(u^{t-1},p(z)^{t-1})} \tag{21.26a}$$

where the right-hand side expression is the ratio of the cost of achieving a period $t-1$ level of utility (u^{t-1}), where utility is a function of the vector of quantities; i.e., $u^{t-1}=f(x^{t-1})$; the price comparison is evaluated at a fixed level of period $t-1$ quantities and s_i^{t-1} are the shares in the total value of expenditure on product i in period $t-1$:

$$s_i^{t-1}=x_i^{t-1}p_i^{t-1}\Big/\sum_{i=1}^{N}x_i^{t-1}p_i^{t-1}$$

21.49 The difference between a Laspeyres formula and the left-hand side of equation (21.26a) is that the price in the numerator of the left-hand side of equation (21.26a) is a predicted price:

$$\hat{p}_i^t\equiv\hat{p}_i^t(z_i^{t-1})=\sum_{k=0}^{K}\beta_k^t z_{ik}^{t-1} \tag{21.26b}$$

or, if a non-comparable replacement is used, then the predicted price adjusts for the difference in quality between the old and new items. That is, the predicted price

$$\hat{p}_i^t\equiv p_i^t-\sum_{i=1}^{N}\beta_k^t(z_{ik}^t-z_{ik}^{t-1}) \tag{21.26c}$$

is the price in period t adjusted for the sum of the changes in each quality characteristic weighted by their coefficients derived from a linear hedonic regression. Note that the summation is over the same i in both periods since replacements are included when an item is missing, and (21.26c) adjusts the prices in period t for quality differences via $\sum_{i=1}^{N}\beta_k^t(z_{ik}^t-z_{ik}^{t-1})$.

21.50 A Paasche lower bound is estimated as:

$$\frac{\sum_{i=1}^{N}x_i^tp_i^t}{\sum_{i=1}^{N}x_i^t\hat{p}_i^{t-1}}=\left[\sum_{i=1}^{N}s_i^t\left(\frac{\hat{p}_i^{t-1}}{p_i^t}\right)\right]^{-1}\leq\frac{C(u^t,p(z)^t)}{C(u^t,p(z)^{t-1})} \tag{21.27a}$$

where $s_i^t=x_i^tp_i^t/\sum_{i=1}^{N}x_i^tp_i^t$ and

$$\hat{p}_i^{t-1}\equiv\sum_{k=0}^{k}\beta_k^{t-1}z_{ik}^t \tag{21.27b}$$

$$\hat{p}_i^{t-1} \equiv p_i^{t-1} + \sum_{k=0}^{N} \beta_k^{t-1}(z_{ik}^t - z_{ik}^{t-1}) \qquad (21.27c)$$

which are the imputation and replacement adjustment, respectively. The latter are the prices in periods $t-1$ adjusted for the sum of the changes in each quality characteristic weighted by their respective coefficients derived from a linear hedonic regression.

21.51 Following from the inequalities in (17.5), where the Laspeyres P_L and Paasche P_P indices form bounds (17.8) on their "true" P_K economic theoretic indices:

$$P_L \le P_K \le P_P \text{ or } P_P \le P_K \le P_L \qquad (21.28)$$

a suitable index is thus a Fisher geometric mean of the Laspeyres P_L and Paasche P_P indices, which incorporate hedonic adjustments for quality differences.

21.52 Thus, the approach based on using superlative and exact hedonic indices first applies the coefficients from hedonic regressions to changes in the characteristics to adjust observed prices for quality changes. Second, it incorporates a weighting system using data on the quantities sold of each model and their characteristics, rather than treating each model as equally important. Finally, it has a direct correspondence to the formulation defined using economic theory.

21.53 Semi-logarithmic hedonic regressions would supply a set of β coefficients suitable for use with the base and current period geometric bounds:

$$\prod_{i=1}^{N}\left(\frac{p_i^t}{\hat{p}_i^{t-1}}\right)^{s_i^t} \le \frac{C(u^t, p(z)^t)}{C(u^t, p(z)^{t-1})} \le \prod_{i=1}^{N}\left(\frac{\hat{p}_i^t}{p_i^{t-1}}\right)^{s_i^{t-1}} \qquad (21.29a)$$

$$\hat{p}_i^{t-1} \equiv \exp\left[\sum_{k=0}^{k} \beta_k^{t-1} z_k^t\right]$$

$$\hat{p}_i^{t} \equiv \exp\left[\sum_{k=0}^{k} \beta_k^{t} z_k^{t-1}\right] \qquad (21.29b)$$

$$\hat{p}_i^{t-1} \equiv p_i^{t-1} \exp\left[\sum_{i=1}^{N} \beta_k^{t-1}(z_{ik}^t - z_{ik}^{t-1})\right]$$

$$\hat{p}_i^{t} \equiv p_i^{t} \exp\left[-\sum_{i=1}^{N} \beta_k^{t}(z_{ik}^t - z_{ik}^{t-1})\right] \qquad (21.29c)$$

21.54 In the inequality (21.29a), the two bounds on the respective theoretical indices have been shown to be brought together. The calculation of such indices is relatively straightforward for matched data, but for unmatched data is no small task. For an example of its application for unmatched comparisons over time, see Silver and Heravi (2002; 2003) and Chapter 7, paragraphs 7.132 to 7.152, and see Kokoski, Moulton and Zieschang (1999) for matched price comparisons across regions of a country.

21.55 Exact hedonic indices can also be defined using the theoretical framework outlined by Diewert (2003a).[29] Recall the basic hedonic equation (21.19). Assume that the price P_k^t is the average price for all the models of type k sold in period t and let q_k^t be the number of units sold of model k in period t. Recall that the number of models in the marketplace during period t is K^t. Assume that there are K models in the marketplace over all T periods in our sample period. If a particular model k is not sold at all during period t, then it will be assumed that P_k^t and q_k^t are both zero. With these conventions in mind, the *total value of consumer purchases during period t* is equal to:

$$\sum_{k=1}^{K} P_k^t q_k^t = \sum_{k=1}^{K} r_t f(z_k) q_k^t \quad \text{for } t=1,\ldots,T \qquad (21.30)$$

21.56 The hedonic sub-utility function f has done all the hard work in the model by converting the utility yielded by model k in period t into a "standard" utility $f(z_k)$ that is cardinally comparable across models. For each model type k, it is only necessary to multiply by the total number of units sold in period t, q_k^t, in order to obtain the *total period* t *market quantity of the hedonic commodity*, Q_t say. This yields:[30]

$$Q_t \equiv \sum_{k=1}^{K} f(z_k) q_k^t \quad \text{for } t=1,\ldots,T \qquad (21.31)$$

21.57 The aggregate price for the hedonic commodity corresponding to Q_t is r_t. Thus in the highly simplified model outlined in paragraphs 21.29 to 21.36, the *aggregate exact period* t *price and quantity* for the hedonic commodity are r_t and Q_t defined by equation (21.31), which can readily be calculated provided that the parameters in the hedonic regression have been estimated and provided that data on quantities sold are available each period q_k^t.[31] Once r_t and Q_t have been determined for $t=1,\ldots,T$, then these aggregate price and quantity estimates for the hedonic commodity can be combined with the aggregate prices and quantities of non-hedonic commodities using normal index number theory. Any of the index number formulae considered in Chapter 17, including Laspeyres, Paasche and Fisher, can be accordingly defined based on the use of quantity information.

21.58 The above illustrates how weighted quality-adjusted price index number formulae might be constructed using data on prices, quantities and characteristics of an item. The method using dummy variables of time, described in paragraphs 21.40 to 21.42, does not require matched data. Appendix 21.1 discusses a weighting system. The use of weighted superlative indices for matched data is outlined above. Weighted superlative indices may also be applied to unmatched data, using a method

[29] The assumptions are quite different from those made by Fixler and Zieschang (1992), who took yet another approach to the construction of exact hedonic indices.

[30] This is a counterpart to the quantity index defined by Muellbauer (1974, p. 988) in one of his hedonic models; see his equation (30). Of course, treating r_t as a price for the hedonic commodity quantity aggregate defined by equation (21.31) can be justified by appealing to Hicks' (1946, pp. 312–313) aggregation theorem, since the model prices $p_k^t = r_t f(z_t)$ all have the common factor of proportionality r_t.

[31] If data are available for the q_k^t, then it is best to run sales-weighted regressions, as discussed in Appendix 21.1. If we do not have complete market data on individual model sales but we do have total sales in each period, then the hedonic regression model can be run using a sample of model prices, and period t sales can be divided by our estimated r_t parameter in order to obtain an estimator for Q_t.

outlined in Chapter 7 and in Silver and Heravi (2001a; 2001b; 2003). But what of unweighted indices, which was the concern of the initial section of this chapter? What correspondence does the unweighted hedonic time dummy index (outlined in paragraphs 21.40 to 21.42), which uses all the data, have to the matched unweighted index number formulae? This is a critical question for product areas where there is a rapid turnover of items. It was suggested above that the time dummy variable method be used instead of the matched method. So how do they differ for unweighted indices? The effect and use of weights is considered in Appendix 21.1.

Unweighted hedonic indices and unweighted matched index number formulae

21.59 Triplett (2002) and Diewert (2003a) have argued that an unweighted geometric mean Jevons index (see equation (20.3)) for matched data gives the same result as a logarithmic dummy variable hedonic index run on the same data. An index from a dummy variable hedonic regression such as equation (21.24), but in double-logarithmic (log-log) form, for matched models can be shown (see Aizcorbe, Corrado and Doms (2001)) to be equal to:

$$\ln(p^t/p^{t-1}) = \sum_{m \in Mt} (\ln p_m^t - Z_m^t)/M^t - \sum_{m \in Mt-1} (\ln p_m^{t-1} - Z_m^{t-1})/M^{t-1} \quad (21.32)$$

where m is the matched sample and Z^t and Z^{t-1} are the quality adjustments to the dummy variables for time in equation (21.24), that is, $\sum_{k=1}^{K}\beta_k z_k^t$. Equation (21.32) is simply the difference between two geometric means of quality-adjusted prices. The sample space $m = M_t = M_{t-1}$ is the same model in each period. Consider the introduction of a new model n introduced in period t with no counterpart in $t-1$ and the demise of an old model o so it has no counterpart in t. So in period t, the sample space M_t is composed of the period t matched items m and the new items n, and in period $t-1$ M_{t-1} is composed of the period $t-1$ matched items m and the old items. Silver and Heravi (2002) have shown the dummy variable hedonic comparison to be:

$$\begin{aligned}\ln(p^t/p^{t-1}) = {} & [m/(m+n)\sum_m(\ln p_m^t - Z_m)/m \\ & + n/(m+n)\sum_n(\ln p_n^t - Z_n)/n] \\ & -[m/(m+o)\sum_m(\ln p_m^{t-1} - Z_m)/m \\ & + o/(m+o)\sum_o(\ln p_o^{t-1} - Z_o)/o] \\ = {} & [m/(m+n)\sum_m(\ln p_m^t - Z_m)/m \\ & - m/(m+o)\sum_m(\ln p_m^{t-1} - Z_m)/m] \\ & + [n/(m+n)\sum_n(\ln p_n^t - Z_n)/n \\ & - o/(m+o)\sum_o(\ln p_o^{t-1} - Z_o)/o] \quad (21.33)\end{aligned}$$

21.60 Consider the second expression in equation (21.33). First there is the price change for m matched observations. This is the change in mean prices of matched models m in period t and $t-1$ adjusted for quality. Note that the weight in period t for this matched component is the proportion of matched observations to all observations in period t, both matched and unmatched new (n). Similarly, for period $t-1$, the matched weight depends on how many matched and unmatched old observations (o) are in the sample. In the last line of equation (21.33) the change is between the unmatched new and the unmatched old mean (quality-adjusted) prices in periods t and $t-1$. Thus, the matched methods can be seen to ignore the last line in equation (21.33) and will thus differ from the hedonic dummy variable approach. It can be seen from equation (21.33) that the hedonic dummy variable approach, in its inclusion of unmatched old and new observations, may possibly differ from a geometric mean of matched prices changes, the extent of any difference depending, in this unweighted formulation, on the proportions of old and new items leaving and entering the sample and on the price changes of old and new items relative to those of matched items. If the market for products is one in which old quality-adjusted prices are unusually low while new quality-adjusted prices are unusually high, then the matched index will understate price changes; see Silver and Heravi (2002) and Berndt, Ling and Kyle (2003) for examples. Different market behaviour will lead to different forms of bias. There is a second way in which the results will differ. Index number formulae provide weights for the price changes. The Carli index, for example, weights each observation equally, while the Dutot index weights each observation according to its relative price in the base period. The Jevons index, with no assumptions as to economic behaviour, weights each observation equally. Silver (2002) has argued, however, that the weight given to each observation in an ordinary least squares regression also depends on the characteristics of the observations, some observations with unusual characteristics having more leverage. In this way, the results from the two approaches may differ even more.

New goods and services

21.61 This section briefly highlights theoretical issues relating to the incorporation of new goods into the index. Practical issues are outlined in Chapter 8, paragraphs 8.36 to 8.60. The term "new goods" will be used here to refer to those that provide a substantial and substantive change in what is provided, as opposed to more of a currently available set of service flows, such as a new model of a car that has a bigger engine. In this latter instance, there is a continuation of a service and production flow, and this may be linked to the service flow and production technology of the existing model. The practical concern with the definition of new goods as against quality changes is that the former cannot be easily linked to existing items as a continuation of an existing resource base and service flow, because of the very nature of their "newness". There are alternative definitions; Oi (1997) directs the problem of defining "new" goods to that of defining a monopoly. If there is no close substitute, the good is new. A monopoly supplier may be able to supply an item with new combinations of the hedonic *z* characteristics resulting from a new technology and have a monopoly power in doing so, but in practice the new good can be linked via the set of hedonic characteristics to the existing goods. In this practical sense, such goods are not considered "new" for the purposes of the manual.

21.62 The terminology adopted here is that used by Merkel (2000) for the measurement of producer price indices, but considered in the context of consumer price indices (CPIs). The aim is to distinguish between *evolutionary* and *revolutionary* goods. Evolutionary goods are replacement or supplementary models which continue to provide a similar service flow, perhaps in new ways or to different degrees. In contrast, revolutionary goods are entirely new goods not closely tied to a previously available good. Although revolutionary goods may satisfy a long-standing consumer need in a novel way, they do not fit into any established CPI item category. Thus, in principle, the underlying theory of what is meant by new goods applies as much to evolutionary as to revolutionary goods. However, the practicalities of index number construction imply a need to consider a new good in terms of something that is not an extension or a modification of an existing one. Evolutionary goods can be incorporated into an index by means of the methods discussed in Chapter 7, even though utility gains from their introduction are ignored. This procedure has further problems. Because the item is by its nature unique, it is unlikely to be incorporated into the sample as a replacement for an existing one. It would be neither comparable nor amenable to explicit adjustments to its price for quality differences with existing goods. Since it is not replacing an item, it does not have an existing weight and its introduction implies a need to reweight the index.

21.63 The main concern regarding the incorporation of new goods into the CPI is the decision on the need and timing for their inclusion. Waiting for a new good to be established or waiting for the rebasing of an index before incorporating new products may lead to errors in the measurement of price changes if the unusual price movements at critical stages in the product life cycles are ignored. There are practical approaches to the early adoption of both evolutionary and revolutionary goods, as outlined in Chapter 8. For evolutionary goods, such strategies include the rebasing of the index, resampling of items and introduction of new goods as directed sample *substitutions*; see Merkel (2000). Also of use are hedonic quality adjustments and indices outlined in Chapter 7, paragraphs 7.103 to 7.109 and 7.153 and 7.158, and paragraphs 21.37 to 21.60, which facilitate the incorporation of evolutionary goods. These goods possess a similar characteristic set to existing goods, but deliver different quantities of these characteristics. Short-run or chained frameworks outlined in paragraphs 7.153 to 7.173 may also be more appropriate for product areas with a high turnover of items. These approaches can incorporate the price change of new goods into the index as soon as prices are available for two successive periods, though issues relating to the proper weighting of such changes may remain.

21.64 For revolutionary goods, however, substitution may not be appropriate. First, revolutionary goods may not be able to be defined within the existing classification systems. Second, they may be primarily produced by a new outlet, which will require extending the sample to such outlets. Third, there will be no previous items to match them against to make a quality adjustment to prices, since by definition they are substantially different from pre-existing goods. Finally, there is no weight to attach to the new outlet or item. Sample *augmentation* is appropriate for revolutionary goods, as opposed to sample substitution for evolutionary goods. It is necessary to bring the new revolutionary goods into the sample in addition to what exists. This may involve extending the classification, the sample of outlets, and the item list within new or existing outlets (Merkel, 2000).

21.65 The second measurement issue with respect to new products is the incorporation of the welfare effect of those products at introduction. The preceding discussion has been concerned with the incorporation of price changes into the index once two successive quotations are available. Yet there is a gain to the consumer when comparing the price in the first of these periods with the price in the period that preceded its introduction *had it existed*. In the context of the CPI, the appropriate period 1 shadow price for the new good is that price that just induces the consumer of the new good to consume zero quantities in the preceding period. This is a hypothetical price. If it is relatively high in the period before the introduction of the good, but the actual price in the period of introduction is much lower, then the introduction of the new good is clearly of some benefit to the consumer. To ignore this benefit, and the change from the virtual price to the actual price in its period of introduction, is to ignore something of the price movements that give rise to expenditure changes.

21.66 The sample augmentation procedures miss the effects on price between the period preceding the introduction of a new good and its introduction. There exist in economic theory and practice the tools for estimating such effects; see Hicks (1940) and Diewert (1980, pp. 498–503). This involves setting a virtual price in the period before introduction. This price is the one at which supply is set to zero. The virtual price is compared with the actual price in the period of introduction and this is used to estimate the welfare gain from the introduction of the good. Hausman (1997) provides some estimates of consumer welfare for the introduction of a new brand of breakfast cereals, Apple-Cinnamon Cheerios. He concludes:

> The correct economic approach to the evaluation of new goods has been known for over fifty years, since Hicks' pioneering contribution. However, it has not been implemented by government statistical agencies, perhaps because of its complications and data requirements. Data are now available. The impact of new goods on consumer welfare appears to be significant according to the demand

estimates of this paper, the CPI for cereal may be too high by about 25 per cent because it does not account for new cereal brands. An estimate this large seems worth worrying about.

21.67 Shapiro and Wilcox (1997b, p. 144) share the same concerns:

> ... the rare new item that delivers services radically different from anything previously available. For example, even the earliest generation of personal computers allowed consumers to undertake tasks that previously would have been prohibitively expensive.
>
> This problem can be solved only by estimating the consumer surplus created by the introduction of each new item. Hausman (1997) argues that this must involve explicit modeling of the demand for each new item. ... Although explicit modeling of demand may be of dubious practicality for widespread implementation in the CPI, strategic application in a few selected cases might be worthwhile.

21.68 The expertise required for such estimates is considerable, and even when applied, is not beyond dispute; see Bresnahan (1997) on this last point. An alternative approach is outlined for the CPI by Balk (2000b), with empirical estimates provided by de Haan (2001), the details being provided in Chapter 8 and Appendix 8.2. While this approach is simpler than that undertaken by Hausman (1997), both require considerable statistical and econometric expertise. The inclusion of such effects on a routine basis is not something being actively considered, even by statistical offices with well-developed systems.[32]

[32] Even if virtual prices were estimated, there would still be problems with including new goods in indices such as the Laspeyres index because of the absence of weights in the base period.

Appendix 21.1 Some econometric issues

1. Hedonic regression estimates are seen in Chapter 7 to have a potential use for quality adjustment to prices. A number of issues arise from the specification and estimation of hedonic regressions, the use of diagnostic statistics, and courses of action when the standard assumptions of ordinary least squares (OLS) are seen to break down. Many of these issues are standard econometric ones and not the subject of this manual. This is not to say they are unimportant. The use of hedonic regressions requires some econometric and statistical expertise, but suitable texts are generally available; see Berndt (1991) – particularly the chapter on hedonic regressions – and Maddala (1988) and Kennedy (1998) amongst many others. Modern statistical and econometric software has adequate diagnostic tests for testing when OLS assumptions break down. There remain, however, some specific aspects which merit attention; these points are over and above the important standard econometric considerations dealt with in econometric texts.

Identification and appropriate estimators

2. Wooldridge (1996, pp. 400–401) has shown on standard econometric grounds that the estimation of supply and demand functions by OLS is biased and this bias carries over to the estimation of the hedonic function. It is first useful to consider estimation issues regarding demand and supply functions. The demand and supply functions are rarely estimated in practice. The more common approach is to estimate *offer* functions, with the marginal price offered by the firm dependent upon chosen attributes (product characteristics) and firm characteristics, and *bid* or value functions, with the marginal prices paid by a consumer dependent upon chosen attributes and consumer characteristics.[33] As noted earlier, the observed prices and quantities are the result of the interaction between structural demand and supply equations and the distributions of producer technologies and consumer tastes; they cannot reveal the parameters of the offer and value functions. Rosen (1974, pp. 50–51) suggested a procedure for determining these parameters. Since these estimates are conditioned on tastes (α) and technologies (τ), the estimation procedure needs to include empirical measures or "proxy variables" of α and τ. For the tastes α of consumers, the empirical counterparts may be socio-demographic and economic variables, which may include age, income, education and geographical region. For technologies τ, variables may include types of technologies, scale and factor prices. First, the hedonic equation is estimated in the normal manner, without these variables, using the best-fitting functional form. This is to represent the price function that consumers and producers face when making their decisions. Then, an implicit marginal price function is computed for each characteristic as $\partial p(z)/\partial zi = \hat{p}_i(z)$, where $\hat{p}(z)$ is the estimated hedonic equation. Bear in mind that in normal demand/supply studies for *products*, the prices are observed in the market. Prices for *characteristics* are unobserved; this first stage is to estimate the parameters from the hedonic regression. The actual values of each z_i bought and sold are then inserted into each implicit marginal price function to yield a numerical value for each characteristic. These marginal values are used in the second stage[34] of estimation as endogenous variables for the estimation of the demand side:

$$\hat{p}_i(z) = F(z_1, \ldots, z_K, \alpha^*) \qquad \text{(A21.1)}$$

where α^* are the proxy variables for tastes; and the supply side:

$$\hat{p}_i(z) = F(z_1, \ldots, z_K, \tau^*) \qquad \text{(A21.2)}$$

where τ^* are the proxy variables for technologies.

The variables τ^* drop out when there is no variation in technologies and $\hat{p}_i(z)$ is an estimate of the offer function. Similarly, the variables α^* drop out when sellers differ and buyers are identical, and cross-section estimates trace out compensated demand functions.

3. Epple (1987) has argued that Rosen's modelling strategy is likely to give rise to inappropriate estimation procedures of the demand and supply parameters. In the hedonic approach to estimating the demand for characteristics, a difficulty arises from the fact that marginal prices are likely to be endogenous – they depend on the amount of each characteristic consumed and must be estimated from the hedonic function rather than observed directly. There are two resulting problems. First, there is an identification problem (see Epple (1987)) because both the marginal price of a characteristic and the inverse bid depend on the levels of characteristics consumed. Second, if important characteristics are unmeasured and they are correlated with measured characteristics, the coefficients of measured characteristics will be biased. This applies to all econometric models, but is particularly relevant to hedonic models; on this point, see Wooldridge (1996, pp. 400–401) in particular. The equilibrium conditions for characteristic prices imply functional relationships among the characteristics of demanders, suppliers and products. This in turn reduces the likelihood that important excluded variables will be uncorrelated with the included variables of the model; see also Bartik (1988) on this point. The bias arises because buyers are differentiated by characteristics (y, α) and sellers by technologies τ. The type of item buyers will purchase is related to (y, α) and the type sellers provide to τ. On the plane of combinations of z transacted, the equilibrium ones chosen may be systematically related; the characteristics of buyers are related to those of sellers. Epple (1987) uses the example of stereo equipment: the higher income of some buyers leads to purchases of high-quality equipment, and the technical competence of sellers leads them to provide it. The consumer and producer characteristics may be correlated.

4. Wooldridge (1996, pp. 400–401) suggests that individual consumer and firm characteristics, such as income, education and input prices, should be used as instruments in estimating hedonic functions. In addition, variables other than a good's characteristics should be included as instruments if they are price-determining, such as geographical location (proximity to ports, good road systems, climate and so on). Communities of economic agents are assumed, within which consumers consume and producers produce for each other at prices that vary across communities for identical goods. Variables of the characteristics of the communities will not in themselves enter the demand and supply equation, but are price-determining for observed prices that are recorded across communities. Tauchen and Witte (2001) provide a systematic investigation of the conditions under which the characteristics of consumers, producers and communities will affect the hedonic parameter estimates for a single regression equation estimated across all communities. A key concern is whether the error term of the hedonic price function represents factors that are unobserved by both the economic agents and the researcher, or only by the researcher. In the latter case, the error term may be correlated with the product attributes; instrumental variable estimation is

[33] These are equivalent to inverse demand (or supply) functions, with the prices dependent upon the quantities demanded (or supplied) and the individual consumer (or producer) characteristics.

[34] This two-stage approach is common in the literature, though Wooldridge (1996) discusses the joint estimation of the hedonic, demand-side and supply-side functions as a system.

required. If the error term is *not* correlated with the product characteristics – preferences are quasi-linear – then a properly specified hedonic regression, including community-specific characteristics or appropriate slope dummies, can be estimated using ordinary least squares. In other cases, depending on the correlation between consumer and producer characteristics, assumptions about the error term and the method of incorporating community characteristics into the regression, instrumental variables, including consumer, producer or community dummy or characteristics, may need to be used.

Functional form

5. Triplett (1987 and 2002) argues that neither classical utility theory nor production theory can specify the functional form of the hedonic function.[35] This point dates back to Rosen (1974, p. 54), who describes the observations as being "... a joint-envelope function and cannot by themselves identify the structure of consumer preferences and producer technologies that generate them". A priori judgements as to what the form should look like may be based on ideas as to how consumers and production technologies respond to price changes. These judgements are difficult to make when the observations are jointly determined by demand and supply factors, but not impossible in rare instances. They are, however, complicated when pricing is with a mark-up, the extent of which may vary over the life cycle of a product. Some tied combinations of characteristics will have higher mark-ups than others. New item introductions are likely to be attracted to these areas of characteristic space, and this will have the effect of increasing supply and thus lowering the mark-up and price; see Cockburn and Anis (1998), Feenstra (1995, p. 647) and Triplett (1987). This again must be taken into account in any a priori reasoning – not an easy or straightforward matter.

6. It may be that in some cases the hedonic function's functional form will be straightforward. For example, prices on the web sites for options for products are often additive. The underlying cost and utility structures are unlikely jointly to generate such linear functions, but the producer or consumer is also paying for the convenience of selling in this way and is willing to bear losses or make gains if the cost or utility at higher values of z are priced lower or worth more than the price set. In general, the data should convey what the functional form should look like; imposing artificial structures simply leads to specification bias. For examples of econometric testing of hedonic functional form, see Cassel and Mendelsohn (1985), Cropper, Deck and McConnell (1988), Rasmussen and Zuehlke (1990), Bode and van Dalén (2001) and Curry, Morgan and Silver (2001).

7. The three forms prevalent in the literature are linear, semi-logarithmic and double-logarithmic (log-log). A number of studies have used econometric tests, in the absence of a clear theoretical statement, to choose between them. There have been a large number of hedonic studies and, as illustrated by Curry, Morgan and Silver (2001), in many of these the quite simple forms do well, at least in terms of the $\bar{R}^2$ presented[36] and the parameters according with a priori reasoning, usually on the consumer side. Of the three popular forms, some are favoured in testing; for example, Murray and Sarantis (1999) favoured the semi-logarithmic form, while others, for example Hoffmann (1998), found that the three functional forms scarcely differed in terms of their explanatory power. That the parameters from these simple forms accord with a priori reasoning, usually from the consumer side, is promising, but researchers should be aware that such matters are not assured. There is much that may happen on the supply side to affect parameter values. Indeed Pakes (2001) has argued that no intuitive sign can be given to the parameters of the variables, since producers may vary their price mark-ups on characteristics in ways that would result in counter-intuitive negative signs on some desirable characteristics.

8. Of the three forms, the semi-logarithmic form has much to commend it. The interpretation of its coefficients is quite straightforward, as proportionate changes in prices arise from a unit change in the value of the characteristic[37] (see Chapter 7, paragraphs 7.39 and 7.40). This is a useful formulation since quality adjustments are usually undertaken by making multiplicative as opposed to additive adjustments.

9. The semi-logarithmic form, unlike the log-log model, can incorporate dummy variables for characteristics which are either present, $z_i = 1$, or not, $z_i = 0$. Furthermore, Diewert (2002e) has argued that it is more likely that the errors from a semi-logarithmic hedonic equation are homoskedastic (have a constant variance) compared to the errors from a linear hedonic equation, since items with very large characteristic values will have high prices and are very likely to have relatively large error terms. On the other hand, models with very small amounts of characteristics will have small prices and small means, and the deviation of a model price from its mean will necessarily be small. Since an assumption of OLS is that the residuals are homoskedastic, the semi-logarithmic equation is preferred to the linear one.

10. More complicated forms are, of course, possible. Simple forms have the virtue of parsimony and allow more efficient estimates to be made for a given sample. However, parsimony is not something to be achieved at the cost of misspecification bias. First, if the hedonic function is estimated across multiple independent markets, then interaction terms are required (see Mendelsohn (1984) for fishing sites). Excluding them is tantamount to omitting variables and inappropriately constraining the estimated coefficients of the regression. Tauchen and Witte (2001) have outlined the particular biases that can arise for such omitted variables in hedonic studies. Second, it may be argued that the functional form should correspond to the aggregator for the index – linear for a Laspeyres index, logarithmic for a geometric Laspeyres index, translogarithmic for a Törnqvist index, and quadratic for a Fisher index (see Chapter 17). As Triplett (2002) notes, however, the purpose of estimating hedonic regressions is to adjust prices for quality differences; imposing a functional form on the data which is inconsistent with the data might create an error in the quality adjustment procedure. Yet, as Diewert (2003a) notes, flexible functional forms encompass these simple forms, the log-log form being a special case of the translog form given in equation (17.42) and the semi-log form being a special case of the semi-log quadratic form given in equation (17.49). If there are a priori reasons

[35] Although Arguea, Haseo and Taylor (1994) propose a linear form on the basis of arbitrage for characteristics, held to be likely in competitive markets, Triplett (2002) argues that this is unlikely to be a realistic scenario in most commodity markets.

[36] While the use of $\bar{R}^2$ as a criterion for deciding between the fit of semi-logarithmic and log-log models has some validity, its use is not advised for comparing linear models with either of these logarithmic formulations, a number of tests being appropriate for such comparisons; see Maddala (1988).

[37] There are two caveats: first, $e^{\hat{\beta}-1}$ is required for the interpretation of the coefficients, where $\hat{\beta}$ is the estimated coefficient. Second, the antilogarithms of the OLS estimated coefficients are not unbiased – the estimation of semi-logarithmic functions as transformed linear regressions requires an adjustment to provide minimum variance unbiased estimates of parameters of the conditional mean. A standard adjustment is to subtract half of the coefficient's squared standard error from the estimated coefficient; see Goldberger (1968) and Teekens and Koerts (1972).

to expect interaction terms for specific characteristics, as illustrated in the example in paragraph 7.99, then these more general forms allow this. The theory of hedonic functions neither dictates the form of the hedonic form nor restricts it.

Changing tastes and technologies

11. The estimates of the coefficients from a hedonic regression may change over time. Some of this change will be attributed to sampling error, especially if multicollinearity is present, as discussed below. But in other cases it may be a genuine reflection of changes in tastes and technologies. If a subset of the estimated coefficients from a hedonic regression is to be used to make a quality adjustment to a non-comparable replacement price, then the use of estimated out-of-date coefficients from some previous period to adjust the prices of the new replacement model may be inappropriate. There is a need to update the indices as regularly as the changes demanded.[38] Estimating hedonic imputation indices is more complicated. Silver (1999), using a simple example, showed how the estimate of quality-adjusted price changes requires a reference basket of characteristics. This is apparent for the hedonic imputation indices in paragraphs 21.37 to 21.60, where separate indices using base and current period characteristics are estimated. A symmetric average of such indices is considered appropriate. A hedonic index based on a time dummy variable implicitly constrained the estimated coefficients from the base and current periods to be the same. Diewert (2003a) formalized the problem of choosing the reference characteristics when comparing prices over time, when the parameters of the hedonic function may themselves be changing over time. He found the results of hedonic indices *not* to be invariant to the choice of reference period characteristic vector set z. He considered the use of a sales-weighted average vector of characteristics, as proposed by Silver (1999), but he notes that over long time periods this may become unrepresentative.[39] Of course, if a chained formulation is used, the weighted averages of characteristics remain reasonably up to date, although chaining has its own pros and cons (see paragraph 17.44 to 17.49 of Chapter 17). A fixed base alternative noted by Diewert (2003a) is to use a Laspeyres type comparison with the base period parameter set, and a Paasche type current period index with the current period parameter set, and take the geometric mean of the two indices for reasons similar to those given in Chapter 15, paragraphs 15.18 to 15.32. The resulting Fisher type index is akin to a geometric mean of Laspeyres and Paasche indices – given in equations (21.26) and (21.27) – based on Feenstra (1995).[40] A feature of the time dummy approach in paragraphs 21.40 to 21.42 is that it implicitly takes a symmetric average of the coefficients by constraining them to be the same. But what if, as is more likely the case, only base period hedonic regression coefficients are available? Since hedonic indices based on a symmetric average of the coefficients are desirable, the "spread" or difference between estimates based on either a current or a reference period characteristic set is an indication of potential bias and estimates of such spread may be undertaken retrospectively. If the spread is large, estimates based on the use of a single period's characteristics set, say the current period, should be treated with caution. More regular updating of the hedonic regressions is likely to reduce spread because the periods being compared will be closer and the characteristics of the items in the periods compared more similar.

Weighting

12. Ordinary least squares estimators implicitly treat each item as being of equal importance, even though some items will have quite substantial sales, while sales of others will be minimal. It is axiomatic that an item with sales of over 5,000 in a month should not be accorded the same influence in the regression estimator as one with a few transactions. Items with very low sales may be at the end of their life cycles or be custom-made. Either way, their (quality-adjusted) prices and price changes may be unusual.[41] Observations with unusual prices should not be allowed unduly to influence the index.[42]

13. The estimation of hedonic regression equations by a weighted least squares (WLS) estimator is preferable. This estimator minimizes the sum of *weighted* squared deviations between the actual prices and the predicted prices from the regression equation, as opposed to ordinary least squares (OLS), which uses an equal weight for each observation. There is a question as to whether to use quantity (volume) or expenditure weights. The use of quantity weights can be supported by considering the nature of their equivalent "price". Such prices are the average (usually the same) price over a number of transactions. The underlying sampling unit is the individual transaction, so there is a sense that the data may be replicated as being composed of, say, 12 individual observations using an OLS estimator, as opposed to a single observation with a weight of 12 using a WLS estimator. Both would yield the same result. Diewert (2002e) has argued on the grounds of representativity that sales values are the appropriate weights. Quantity weighting gives too little weight to models with high prices and too much weight to cheap models that have relatively low amounts of useful characteristics. The need to equate the weights with relative expenditure or sales value arises from a prime concern with index numbers: that they serve to decompose changes in value into their price and quantity components. Silver (2002) has shown that a WLS estimator using value weights will not necessarily give each observation a weight equal to its relative value. The estimator will give more weight to those observations with high leverage effects and residuals. Observations with values of characteristics with large deviations from their means, say very old or new models, have relatively high leverage. New and old models are likely to be priced at quite different prices than those predicted from the hedonic regression, even after taking into account their different characteristics. Such prices result, for example, from a pricing strategy designed to skim segments of the market willing to pay a premium for a new model, or from a strategy to charge relatively low prices for an old model to dump it to make way for a new one. In such cases, the influence these models have on deriving the estimated coefficients will be over and above that attributable to their value weights.

[38] Adjusting the base versus the current period price entails different data demands; see Chapter 7, paragraph 7.49.

[39] Other averages may of course be proposed; for example, the needs of an index representative of the "typical" establishment would be better met by a trimmed mean or median.

[40] Diewert (2002e) also suggests matching items where possible, and using hedonic regressions to impute the prices of the missing old and new ones. Different forms of weighting systems, including superlative ones, can be applied to this set of price data in each period for both matched and unmatched data.

[41] Such observations would have higher variances of their error terms, leading to imprecise parameter estimates. This would argue for the use of weighted least squares estimators with quantity sold as the weight. This is one of the standard treatments for heteroskedastic errors; see Berndt (1991).

[42] Silver and Heravi (2002) show that old items have above-average leverage effects and below-average residuals. Not only are they different, but they exert undue influence for their size (number of observations). See Berndt, Ling and Kyle (2003), Cockburn and Anis (1998) and Silver and Heravi (2002) for examples.

Silver (2002) suggests that leverage effects should be calculated for each observation, and those with high leverage and low weights should be deleted, and the regression rerun. Thus, while quantity or value weights are preferable to no weights (i.e., OLS), value weights are more appropriate than quantity ones and, even so, account should be taken of observations with undue influence.

14. Diewert (2002e) has also considered the issue of weighting with respect to the dummy time hedonic indices outlined in paragraphs 21.40 to 21.42. The use of WLS by value involves weights being applied to observations in both periods. However, if, for example, there is high inflation then the sales values for a model in the current period will generally be larger than those of the corresponding model in the base period and the assumption of homoskedastic residuals is unlikely to be met. Diewert (2002e) suggests the use of expenditure *shares* in each period, as opposed to values, as weights for WLS for time dummy hedonic indices. He also suggests that an average of expenditure shares in the periods being compared be used for matched models.

15. Data on sales are not always available for weights, but the major selling items can generally be identified. In such cases, it is important to restrict the number of observations of items with relatively low sales, the extent of the restriction depending on the number of observations and the skewness of the sales distribution. In some cases, items with few sales provide the variability necessary for efficient estimates of the regression equation. In other cases, their low sales may be due to factors that make them unrepresentative of the hedonic surface, their residuals being unusually high. An example is low-selling models about to be dumped to make way for new models. Unweighted regressions may thus suffer from a sampling problem – even if the prices are perfectly quality adjusted, the index can be biased because it is unduly influenced by low-selling items with unrepresentative price–characteristic relationships. In the absence of weights, regression diagnostics have a role to play in helping to determine whether the undue variance in some observations belongs to such unusual low-selling items.[43]

Multicollinearity

16. There are a priori reasons to expect, for some products, that the variation in the value of one quality characteristic is not independent of one quality characteristic or a linear combination of more than one such characteristic. As a result, parameter estimates will be unbiased yet imprecise. To illustrate this, a plot of the confidence interval for one parameter estimate against another collinear one is often described as elliptical, since the combinations of possible values they may take can easily drift from, say, high values of β_1 and low values of β_2 to high values of β_2 and low values of β_1. Since the sample size for the estimates is effectively reduced, additions to and deletions from the sample may affect the parameter estimates more than would be expected. These are standard statistical issues, dealt with by Maddala (1988) and Kennedy (1998). In a hedonic regression, multicollinearity might be expected, as some characteristics may be technologically tied to others. Producers including one characteristic may need to include others for the product to work, while consumer purchasing, say, an up-market brand may expect a certain bundle of features to come with it. Triplett (2002) argues strongly for the researcher to be aware of the features of the product and the consumer market. There are standard, though not completely reliable, indicators of multicollinearity (such as variance inflation factors), but an exploration of its nature is greatly aided by an understanding of the market along with exploration of the effects of including and excluding individual variables on the signs and coefficients and on other diagnostic test statistics; see Maddala (1988).[44]

17. If a subset of the estimated coefficients from a hedonic regression is to be used to quality-adjust a non-comparable replacement price, and if there is multicollinearity between variables in this subset and other independent variables, then the estimates of the coefficients to be used for the adjustment will be imprecise. The multicollinearity effectively reduces the sample size, and some of the effects of the variables in the subset may be wrongly ascribed to the other independent variables. The extent of this error will be determined by the strength of the multiple correlation coefficient between all such "independent" variables (the multicollinearity), the standard error or fit of the regression, the dispersion of the independent variable concerned and the sample size. These all affect the precision of the estimates since they are components in the standard error of the *t*-statistics. Even if multicollinearity is expected to be quite high, large sample sizes and a well-fitting model may reduce the standard errors on the *t*-statistics to acceptable levels. If multicollinearity is expected to be severe, the predicted value for an item's price may be computed using the whole regression and an adjustment made using this predicted value, as explained in Chapter 17, paragraphs 17.103 to 17.109. There is a sense in which it does not matter whether the variation that, for example, should have been attributed to β_1 was wrongly attributed to β_2, or vice versa if the predicted price based on both β_1 and β_2 is used.

Omitted variable bias

18. The exclusion of tastes, technology and community characteristics has already been discussed. The concern here is with product characteristics. Consider again the use of a subset of the estimated coefficients from a hedonic regression to quality-adjust a non-comparable replacement price. It is well established that multicollinearity of omitted variables with included variables leads to bias in the estimates of the coefficients of included ones. If omitted variables are independent of the included variables, then the estimates of the coefficients on the included variables are unbiased. This is acceptable in the present instance, the only caveat being that the quality adjustment for the replacement item may also require an adjustment for these omitted variables and this, as noted by Triplett (2002), has to be undertaken using a separate method and data. But what if the omitted variable is multicollinear with a subset of included variables which are to be used to quality-adjust a non-comparable item? In this case, the coefficient of the subset of the included variables may wrongly pick up some of the effects of the omitted variables. The subset of included variables will be used to quality-adjust prices for items which differ only with regard to this subset, and the price comparison will be biased if the characteristics of included and

[43] A less formal procedure is to take the standardized residuals from the regression and plot them against model characteristics that may denote low sales, such as certain brands (makes) or vintage (if not directly incorporated), or some technical feature which makes it unlikely that the item is being bought in quantity. Higher variances may be apparent from the scatter plot. If certain features are expected to have, on average, low sales, but seem to have high variances, leverages and residuals (see Silver and Heravi (2002)), a case exists for at least down-playing their influence. Bode and van Dalén (2001) use formal statistical criteria to decide between different weighting systems and compare the results of OLS and WLS, finding, as with Ioannidis and Silver (1999), that different results can arise.

[44] Triplett (2002) stresses the point that $\bar{R}_2$ alone is insufficient for this purpose.

omitted variables have different price changes. For hedonic indices using a dummy time trend, the estimates of quality-adjusted price changes will suffer from a similar bias if omitted variables that are multicollinear with the time change are excluded from the regression. What are picked up as quality-adjusted price changes over time may, in part, be changes attributable to the prices of these excluded variables. This happens when the prices of the omitted characteristics follow a different trend. Such effects are most likely when there are gradual improvements in the quality of items, such as the reliability and safety of consumer durables,[45] which are difficult to measure, at least for the sample of items in real time. The quality-adjusted price changes will thus overstate price changes in such instances.

[45] There are, of course, some commodity areas, such as airline comfort, which have been argued to have overall patterns of decreasing quality.