21. Quality Change and Hedonics

21.1 Chapters 15 to 20 cover theoretical issues relating to the choice of index number formulas and are based on a simplifying assumption: that the aggregation was over the same matched $i = 1,\ldots,n$ items in the two periods being compared. This meets the needs of the discussion of alternative index number formulas, since a measure of price change between two periods requires the quality of each item to remain the same. The practical compilation of CPIs involves defining the quality specification of a sample of items in an initial period and monitoring the prices of this matched sample over time, so that only pure price changes are measured, not price changes tainted by changes in quality. In practice, this matching becomes imperfect. The quality of what is sold *does* change and, furthermore, new goods (and services) appear on the market that the matched sampling ignores. The relative price changes of these new goods may differ from those of the existing ones, leading to bias in the index if they are excluded. In this chapter, a theoretical framework is outlined that extends the definition of items to include their quality characteristics. The focus of the chapter is on the *economic* theory of the market for quality characteristics and its practical manifestation in hedonic regression outlined in Chapter 7. This chapter provides a *background* for the more practical issues relating to quality adjustments in Chapter 7 and item substitution in Chapter 8.

A. New and Disappearing Items and Quality Change: Introduction

21.2 The assumption in the previous chapters was that the same set of items was being compared in each period. Such a set can be considered as a sample from all the matched items available in periods 0 and t—the *intersection universe*, which includes only matched items. [1] Yet for many products old items disappear and new items appear. Constraining the sample to be drawn from this intersection universe is unrealistic. Outlets may sell an item in period 0, but it may not be sold in subsequent periods t.[2] New items may be introduced after period 0 that cannot be compared with a corresponding item in period 0. These items may be variants of the old existing one, or provide totally new services that cannot be directly compared with anything that previously existed. This universe of all items in periods 0 and t is the dynamic *double universe.*

21.3 There is a third universe from which prices might be sampled: a *replacement* universe. The prices of items in period 0 are first determined, and then their prices are monitored in subsequent periods. If the item is discontinued and there are no longer prices to record for a particular item, prices of a comparable replacement item may be used to continue the series of prices. This universe is a *replacement universe* that starts with the base-period universe, but it also includes one-to-one replacements when an item from the sample in the base period is missing in the current period.

21.4 When a comparable replacement is unavailable, a non-comparable one may be

[1]The terminology is credited to Dalén (1998a), see also Appendix 8.1.

[2]Its absence may be temporary, being a seasonal item, and specific issues and methods for such temporarily unavailable items are considered in Chapter 9. The concern here is with items that disappear permanently.

selected. In this case, an explicit adjustment has to be made to the price of either the old or the replacement item for the quality difference. Since the replacement is of a different quality to the old item, it is likely to have a different price basis. Alternatively, assumptions may be made so that the price change of the old item (had it continued to exist) follows those of other items, keeping to the matched universe. In this second case, an implicit adjustment is being made for quality changes, so that the difference in price changes for the group and the old item (had it continued to exist) is equivalent to their quality differences.[3] What is stressed here is that the problem of missing items is the problem of adjusting prices for quality differences.

21.5 Three practical problems emerge. First, is the problem of explicit quality adjustment between a replacement and old item. The item is no longer sold, a replacement is found that is not strictly comparable in quality, the differences in quality are identified, and a price has to be put on these differences if the series of prices for the new replacement item are to be used to continue those of the old series.

21.6 Second, in markets where the turnover of items is high, the sample space selected from the matched universe is going to become increasingly unrepresentative of the dynamic universe, as argued in detail in Chapter 8. Even the replacement universe may be inappropriate, as it will be made of series carrying with them quality adjustments in each period whose overall accuracy, given the rapidly changing technology, may be tenuous. In such cases, it may be that prices are no longer collected from a matched sample but from a sample of the main items available in each period even though they are of a different quality. A comparison between the average prices of such items would be biased if, say, the quality of the items was improving. The need for, and details of, mechanisms to remove the effects of such changes from the average price comparisons were discussed in some detail in Chapter 7, Section G.

21.7 Finally, there is the problem of new and disappearing goods and services—when the new item is not a variant of the old but provides a completely new service. It is not possible to use it as a replacement for an old item by adjusting a price for the quality differential because what it provides is, by definition, something new.

21.8 There are a number of approaches to quality adjustment, and these are considered in Chapter 7. One of the approaches is to make explicit adjustments to prices for the quality difference between the old and replacement item using the coefficients from hedonic regression equations. *Hedonic regressions* are regressions of the prices of individual models of a product on their characteristics—for example, the prices of television sets on screen size, stereo sound, and text retrieval. The coefficients on such variables provide estimates of the monetary values of different quantifiable characteristics of the product. They can be used to adjust the price of a noncomparable replacement item for quality differences compared with the old item—for example, the replacement television set may have text-retrieval facilities that the previous version did not. Yet, it is important that a clear understanding exists of the meaning of such estimated coefficients if they are to be used for quality adjustment,

[3]Such methods and their assumptions are outlined in detail in Chapter 15.

especially given that their use is being promoted.[4] To understand what these estimated parameters mean, it is first necessary to conceive of products as aggregates of their characteristics because, unlike items, characteristics have no separate prices attached to them. The price of the item is the price of a "tied" bundle of characteristics. One must also consider what determines the prices of these characteristics. Economic theory points toward examining demand and supply factors (Sections B.2 and B.3) and the interaction of the two to determine an equilibrium price (Section B.4). Having developed the analytical framework for such prices, it is then necessary to see what interpretation the economic theoretic framework allows us to put on these calculated coefficients (Section B.5). It will be seen that unless there is uniformity of buyers' tastes or 'technologies, an identification problem prevents an unambiguous supply or demand interpretation. Borrowing a framework by Diewert (2002d), a demand-side interpretation that assumes firms are competitive price takers is provided, which, under this user-value approach, shows the assumptions required to generate such meaningful coefficients (Section B6). All of the aforementioned analysis assumes competitive behavior, an assumption which will be relaxed in Section B7.

21.9 Chapter 7, Section G recommends two main approaches for handling products with rapid turnover of items. If the sample in period 0 is soon outdated, the matched universe, with even one-on-one replacements, can become increasingly unrepresentative of the double universe, and repeated sampling from the double universe is required. In this case either chained indices are advised, as in Chapter 7, Section G.3, or one of a number of *hedonic indices*, described in Chapter 7, Section G.2. Such indices differ from the use of hedonic regression equations for adjusting prices for quality differences for a missing item. These indices use hedonic regressions, say, by including a dummy variable for time on the right hand side of the equation, to estimate the quality-adjusted price change, as outlined below in Section C and in Chapter 7. An understanding of hedonic regression equations requires that the economic theory of consumer price indices, outlined in Chapter 17, be developed to include goods that can be defined in terms of tied bundles of their characteristics. *Theoretical* consumer (cost of living) *price indices* are defined that include changes in the prices of characteristics. Yet, as with the theoretical consumer price indices for goods considered in Chapter 17, there are many formulations that hedonic indices can take, and analogous issues and formulas arise here when discussing alternative approaches in Sections C.3–C.6.

21.10 The estimation of hedonic regressions and the testing of their statistical properties are facilitated by the availability of user-friendly, yet powerful, statistical and econometric software. There are many standard issues in the estimation of regression equations, which can be examined by the diagnostics tests available in such software, as discussed in Kennedy (2003) and Maddala (1988). However, there are issues on functional form, the use of weighted least squares estimators, and specifications that are quite specific to the estimation of hedonic equations. While many of these are taken up in Chapter 7, where an illustration is provided, Appendix 21.1 considers some of the theoretical issues. For additional material on these issues, see Gordon (1990), Griliches (1990), and Triplett (1990).

21.11 Finally, in Section D, economic theory will be used to advise on the problem of new and disappearing goods and services. This problem arises where differences between existing

[4] See Boskin (1996; 1998) and Schultze and Mackie (2002) on this point.

goods and services and the new goods and services are substantive and cannot be meaningfully compared with an old item, even with a quality adjustment. The economic theory of reservation prices will be considered and some issues about its practical implementation will be discussed.

B. Hedonic Prices and Implicit Markets

B.1 Items as tied bundles of characteristics

21.12 A *hedonic regression* is a regression equation that relates the prices of items, p to the quantities of characteristics, given by the vector $z = (z_1, z_2, \ldots, z_n)$, that is,

$$p(z) = p(z_1, z_2, \ldots, z_n), \tag{21.1}$$

where the items are defined in terms of varying amounts of their characteristics. In practice, what will be observed for each item (or variant of the product) is its price, a set of its characteristics, and possibly the quantity and thus, the value sold. Empirical work in this area has been concerned with two issues: estimating how the price of an item changes as a result of unit changes in each characteristic—that is, the estimated coefficients of equation (21.1)—and estimating the demand and supply functions for each characteristic. The depiction of an item as a basket of characteristics, each characteristic having it own implicit (shadow) price, requires in turn the specification of a market for such characteristics, since prices result from the workings of markets. Houthakker (1952), Becker (1965), Lancaster (1966), and Muth (1966) have identified the demand for items in terms of their characteristics. The sale of an item is the sale of a tied bundle of characteristics to consumers, whose economic behavior in choosing between items is depicted as one of choosing between bundles of characteristics.[5] However, Rosen (1974) further developed the analysis by providing a structural market framework in terms of both producers and consumers. There are two sides: demand and supply. How much of each characteristic is supplied and consumed is determined by the interaction of the demand for characteristics by consumers and the supply of characteristics by producers. These are considered in turn.

B.2 Consumer or demand side

21.13 Figure 8.1 in Triplett (1987, p. 634) presents a simplified version of the characteristic space between two characteristics. This figure is reproduced as Figure 21.1 below. The hedonic surfaces denoted by p_1 and p_2 in that figure trace out all the combinations of the two characteristics z_1 and z_2 that can be purchased at prices p_1 and p_2. An indifference curve q_j^* maps the combinations of z_1 and z_2 that the consumer is indifferent against purchasing; that is, the consumer will derive the same utility from any point on the curve. The tangency of q_j^* with p_1 at A is the solution to the utility-maximization problem for a given budget (price p_1) and tastes (reflected in q_j^*).

21.14 The slope of the hedonic surface is the marginal cost of acquiring the combination of

[5] Consumers are typically assumed to have preferences over alternative combinations of characteristics that give rise to continuously differentiable price functions. However, for some models, the price functions are piecewise linear and hence continuous but not differentiable; e.g., see Lancaster (1971) or Gorman (1980).

characteristics, and the slope of the utility function is the marginal utility gained from their purchase. The tangency at *A* is the utility-maximizing combination of characteristics to be purchased at that price. If consumers purchased any other combination of characteristics in the space of Figure 21.1, it would either cost them more to do so or lead to a lower level of utility. Position A', for example, has more of both z_1 and z_2, and the consumer receives a higher level of utility being on q_j, but the consumer also has to have a higher budget and pays p_2 for being there. Note that the hedonic surface depicted here is nonlinear, so that relative characteristic prices are not fixed. The consumer with tastes q_k* chooses characteristic set *B* at p_1. Thus, the data observed in the market depends on the set of tastes. Triplett (2004) has argued that if tastes were all the same, then only one model of a personal computer would be purchased. But in the real world more than one model does exist, reflecting heterogeneous tastes and income levels. Rosen (1974) shows that of all the characteristic combinations and prices at which they may be offered, the hedonic surface traces out an envelope[6] of tangencies including those on q_j* and q_k* on p_1 in Figure 21.1. This envelope is simply a description of the locus of the points chosen. Since rational consumers who optimize are assumed, these are the points that will be observed in the market and are thus, used to estimate the hedonic regression. Alternative z points on the same indifference curve will allow the relative price of z_1 to z_2 to be determined. However, observed data are likely to result from a the locus of points on an expansion paths such as $A\,A'$. There may be expansion paths for consumers with different income levels and tastes, such as *B*, and this may give rise to conflicting valuations, so that the overall parameter estimates determined by the regression from transactions observed in the market are an amalgam of such data. And of course this would just be a reflection of the reality of economic life. What arises from this exposition is the fact that the form of the hedonic function is determined in part by the distribution of buyers and their tastes in the market.

21.15 The exposition is now formalized to include parameters for tastes and a numeraire product[7] against which combinations of other aggregates are selected following Rosen (1974). The hedonic function $p(z)$ describes variation in the market price of the items in terms of their characteristics. The consumer purchase decision is assumed to be based on utility-maximization behavior, the utility function being given by $U(z, x;\alpha)$, where x is a numeraire product, the maximization of utility being subject to a budget constraint given by income y measured as $y = x + p(z)$ (the amount spent on the numeraire product and the hedonic products), and α is a vector of the features of the individual consumer that describes their tastes. Consumers maximize their utility by selecting a combination of quantities of x and characteristics z subject to a budget constraint. The market is assumed to be competitive and consumers are described as price takers, they purchase only the one item, so their purchase decision does not influence the market price. The price they pay for a combination of characteristics, vector z, is given by $p(z)$. Since they are optimizing consumers the combination chosen is such that

[6]An envelope is more formally defined by letting $f(x,y,k) = 0$ be an implicit function of x and y. The form of the function is assumed to depend on k, the tastes in this case. A different curve corresponds to each value of k in the xy plane. The envelope of this family of curves is itself a curve with the property that it is tangent to each member of the family. The equation of the envelope is obtained by taking the partial derivative of f(x,y,k) with respect to k and eliminating k from the two equations $f(x,y,k) = 0$ and $f_k(x,y,k) = 0$. (See Osgood, 1925.)

[7]The numeraire product represents all other goods and services consumed—it represents the normal nonhedonic products. The price of x is set equal to unity; $p(z)$ and income are measured in these units.

$$[\partial U(z, y - p(z);\alpha) / \partial z_i] / [\partial U(z, y - p(z);\alpha) / \partial x] = \partial p(z) / \partial z_i \equiv p_i(z), \qquad (21.2)$$

where $\partial p(z) / \partial z_i$ is the first derivative of the hedonic function in equation (21.1) with respect to each z characteristic. The coefficients of the hedonic function are equal to their shadow price p_i, which measure the utility derived from that characteristic relative to the numeraire good for given budgets and tastes.

21.16 A *value function* θ can be defined as the value of expenditure a consumer with tastes α is willing to pay for alternative values of z at a given utility u and income y represented by $\theta(z;u,y,\alpha)$. It defines a family of indifference curves relating the z_i to foregone x, 'money'. For individual characteristics z_i, θ is the marginal rate of substitution between z_i and money, or the implicit marginal valuation the consumer with tastes α puts on z_i at a given utility level and income. It is an indication of the reservation demand price[8] for additional units of z_i.[9] The price in the market is $p(z)$, and utility is maximized when $\theta(z;u,y,\alpha) = p(z)$, that is the purchase takes place where the surface of the indifference curve θ is tangent to the hedonic price surface. If different buyers have different value functions (tastes), some will buy more of a characteristic than others for a given price function, as illustrated in Figure 21.1.

21.17 The joint distribution function of tastes and income sets out a family of value functions, each of which, when tangential to the price function, depicts a purchase and simultaneously defines the price function whose envelope is the market hedonic price function. The points of purchase traced out by the hedonic function thus, depend on the budget of the individual and the tastes of the individual consumer purchasing an individual set of characteristics. If demand functions are to be traced out, the joint probability distribution of consumers with particular budgets and tastes occurring in the market needs to be specified, that is, $F(y, \alpha)$. This function, along with equation (21.1) ,allows the demand equations to be represented for each characteristic.

B.3 Producer or supply side.

21.18 Again referring to Triplett's (1987) Figure 8.1, it also shows the production side. In Chapter 17, Section B.1, a revenue-maximizing producer was considered whose revenue-maximization problem was given by equation (17.1)[10]:

$$R(p,v) \equiv \max_q \left[\sum_{n=1}^{N} p_n q_n : q \text{ belongs to } S(v) \right], \qquad (21.3)$$

where $R(p,v)$ is the maximum value of output, $\sum_{n=1}^{N} p_n q_n$, that the establishment can produce, given that it faces the vector of output prices p and given that the vector of inputs v is available for use, using the period t technology. Figure 17.1 illustrated in goods-space how the producer would choose between different combinations of outputs, q_1 and q_2. In Figure

[8]This is the hypothetical price that makes the demand for the characteristic equal to zero that is, it is the price that, when inserted into the demand function, sets demand to zero.

[9]The utility function is assumed strictly concave so that θ is concave in z, and the value function is increasing in z_i at a decreasing rate.

[10]The time superscripts are not relevant in this context.

21.1, the characteristics-space problem is analogous to the goods-space one with producers choosing here between combinations of z_1 and z_2 to produce for a particular level of technology and inputs $S(v)$. For a particular producer with level of inputs and technology S^*_G facing a price surface p_1, the optimal production combination is at *A*. However, a different producer with technology and inputs S^*_H facing a price surface p_1 would produce at *B*. At these points, the marginal cost of z_1 with respect to z_2 is equal to its marginal price from the hedonic surface as depicted by the tangency of the point. Production under these circumstances at any other combination would not be optimal. The envelope of tangencies such as S^*_G and S^*_H trace out the production decisions that would be observed in the market from optimizing, price-taking producers and be used as data for estimating the hedonic regressions. The hedonic function can be seen to be determined, in part, by the distribution of technologies of producers, including their output scale.

21.19 Rosen (1974) formalizes the producer side, whereby price-taking producers are assumed to have cost functions described by $C(Q, z; \tau)$[11] where $Q = Q(z)$ is the output scale, that is, the number of units produced by an establishment offering specifications of an item with characteristics *z*. They have to decide which items to produce, that is, which package of *z* to produce. The solution for each producer is to choose the output that minimizes costs given its own technology: the output combinations each producer can produce with given input costs using its factors of production and factor prices the technology. The cost function includes τ, equivalent to $S(v)$ above, a vector of the technology and inputs of each producer. It is the variation in τ across producers that distinguishes producer *A*'s decision about which combination of *z* to produce from that of producer *B* in Figure 21.1. Producers are optimizers who seek to maximize profits given by

$$Q\,p(z) - C(Q,z;\tau) \tag{21.4}$$

by selecting *Q* and *z* optimally. The supplying market is assumed to be competitive, and producers are price takers so the producers cannot influence price by their production decision. Their decision about how much to produce of each *z* is determined by the price of *z*, assuming that the producer can vary *Q* and *z* in the short run.[12] Dividing equation (21.4) by *Q* and setting the resulting expression equal to zero, the first-order profit-maximizing conditions are given by

$$\frac{\partial p}{\partial z_i} = p_i = \frac{C_{z_i}(Q, z; \tau)}{Q} \tag{21.5}$$

where $p = p(z_1, z_2, \ldots, z_n)$ as in equation (21.1)

[11]The cost function is assumed to be convex with no indivisibilities. The marginal cost of producing one more item of a given combination of characteristics is assumed to be positive and increasing, and, similarly, the marginal cost of increasing production of each component characteristic is positive and nondecreasing.

[12]Rosen (1974) considered two other supply characterizations: the short run in which only *Q* is variable, and a long run in which plants can be added and retired. The determination of equilibrium supply and demand is not straightforward. A function $p(z)$ is required such that market demand for all *z* will equate to market supply and clear the market. But demand and supply depend on the whole $p(z)$, since any adjustment to prices to equate demand and supply for one combination of items will induce substitutions and changes for others. Rosen (1974, pp. 44–48) discusses this in some detail.

21.20 The *marginal unit revenue* from producing characteristic z_i is given by its shadow price in the price function and its marginal cost of production. In the producer case, a knowledge of the probability distribution of the technologies of firms, $G(\tau)$, is necessary if the overall quantity supplied of items with given characteristic sets are to be revealed. Since it is a profit-maximization problem to select the optimal combination of characteristics to produce, marginal revenue from the additional attributes must equal their marginal cost of production per unit sold. Quantities are produced up to the point where unit revenues $p(z)$ equal marginal production costs, evaluated at the optimum bundle of characteristics supplied.

21.21 While for consumers a *value function* was considered, producers require an *offer function* $\phi(z;\pi,\tau)$. The offer price is the price the seller is willing to accept for various designs at constant profit level π, when quantities produced are optimally chosen, while $p(z)$ is the maximum price obtainable from those models in the market. Producer equilibrium is characterized by a tangency between a profit characteristics indifference surface and the market characteristics price surface, where $p_i(z_i) = \phi_{z_i}(z;\pi,\tau)$ and $p\ (z\ ) = \phi_z(z;\pi,\tau)$. Since there is a distribution of technologies $G(\tau)$, the producer equilibrium is characterized by a family of offer functions that envelop the market hedonic price function. The varying τ will depend on different factor prices for items produced in different countries, multiproduct firms with economies of scale, and differences in the technology, whether the quality of capital, labor, or intermediate inputs and their organization. Different values of τ will define a family of production surfaces.

B.4 Equilibrium

21.22 The theoretical framework first defined each item as a point on a plane of several dimensions made up by the z_1, z_2, ….,z_n quality characteristics; each item was a combination of values z_1, z_2, ….,z_n.. If only two characteristics defined the item, then each point in the positive space of Figure 21.1 would define an item. The characteristics were not bought individually but as bundles of characteristics tied together to make up an item. It was assumed that the markets were differentiated so that there was a wide range of choices to be made.[13] The market was also assumed to be perfectly competitive with consumers and producers as price takers undertaking optimizing behavior to decide which items (tied sets of characteristics) to buy and sell. Competitive markets in characteristics and optimizing behavior are assumed so that the quantity demanded of characteristics z must equal the quantity supplied. It has been shown that consumers' and producers' choices or "locations" on the plane will be dictated by consumer tastes and producer technology. Tauchen and Witte (2001, p. 4) show that the hedonic price function will differ across markets in accordance with the means and variances (and in some cases also higher moments) of the distributions of household and firm characteristics.

21.23 Rosen (1974, p. 44) notes that a buyer and seller are perfectly matched when their respective value and offer functions are tangential. The common gradient at that point is given by the gradient of the market-clearing implicit price function $p(z)$. The consumption

[13]In order to ensure that choices among combinations of z are continuous, assume further that p(z) possesses continuous first order derivatives.

and production decisions were seen in the value and offer functions to be jointly determined, for given $p(z)$, by $F(y, \alpha)$ and $G(\tau)$. In competitive markets there is a simultaneity in the determination of the hedonic equation, since the distribution of $F(y, \alpha)$ and $G(\tau)$ help determined the quantities demanded and supplied and also the slope of the function. Although the decisions made by consumers and producers are as price takers, the prices taken are those from the hedonic function. There is a sense in which the hedonic function and its shadow prices emerge from the operations of the market. The product markets implicitly reveal the hedonic function. Since consumers and producers are optimizers in competitive markets, the hedonic function, in principle, gives the minimum price of any bundle of characteristics. Given all of this, Rosen (1974, p. 44) asked: what do hedonic prices mean?

B.5 What do hedonic prices mean?

21.24 It would be convenient if, for CPI construction, the estimated coefficients from hedonic regressions were estimates of the marginal utility based on a characteristic or user value from a characteristic. But theory tells us that this is not the case and that the interpretation is not clear.

21.25 There was an erroneous perception in the 1960s that the coefficients from hedonic methods represented user-values as opposed to resource-costs. Rosen (1974), as has been shown, found that hedonic coefficients generally reflect both user-values and resource-costs; both supply and demand situations. The ratios of these coefficients may reflect consumers' marginal rates of substitution or producers' marginal rates of substitution (transformation) for characteristics. There is what is referred to in econometrics as an "identification"' problem in which the observed prices and quantities are jointly determined by supply and demand considerations, and their underlying effects cannot be separated. The data collected on prices jointly arise from variations in demand by different consumers with different tastes and preferences, and from variations in supply by producers with different technologies.

21.26 First, it is necessary to come to terms with this simultaneity problem. Hedonic regressions are an increasingly important analytical tool, one implicitly promoted by the attention given to it in this *Manual* but also promoted in separate manuals by organizations such as the OECD (see Triplett, 2004), and Eurostat (2001), and widely used by the U.S. Bureau of Labor Statistics (Kokoski, Waehrer, and Rozaklis, 2001, and Moulton, 2001b). So how do economists writing on the subject shrug their intellectual shoulders in the light of these findings?

21.27 Rosen (1974, p. 43) refers to the hedonic function as "..a joint envelope of a family of value functions and another family of offer functions. An envelope function by itself reveals nothing about the underlying members that generate it; and they in turn constitute the generating structure of the observations."

21.28 Griliches (1988, p. 120) notes the following:

> My own view is that what the hedonic approach tries to do is to estimate aspects of the budget constraint facing consumers, allowing thereby the estimation of "missing" prices when quality changes. It is not in the business of estimating utility functions

> *per se*, though it can also be useful for these purposes….what is being estimated is the actual locus of intersection of the demand curves of different consumers with varying tastes and the supply curves of different producers with possible varying technologies of production. One is unlikely, therefore to be able to recover the underlying utility and cost functions from such data alone, except in very special circumstances.

21.29 Triplett (1987) states, "It is well-established—but still not widely understood—that the form of $h(\cdot)$ [the hedonic function] cannot be derived from the form of $Q(\cdot)$ and $t(\cdot)$ [utility and production functions], nor does $h(\cdot)$ represent a "reduced form" of supply and demand functions derived from $Q(\cdot)$ and $t(\cdot)$."

21.30 Diewert (2003, p. 320) with his focus on the consumer side, says;

> Thus, I am following Muellbauer's (1974, p. 977) example where he says that his "approach is unashamedly one-sided; only the demand side is treated…Its subject matter is therefore rather different from that of the recent paper by Sherwin Rosen. The supply side and simultaneity problems which may arise are ignored."

Diewert (2003) has also considered the theoretical CPI indices with a focus only on consumers' valuations, giving them precedence. In Section B.6 this framework is outlined, which allows a more straightforward development of the theory of hedonic index numbers for CPIs.

21.31 Second, Rosen's theoretical framework allows the conditions to be considered under which the hedonic coefficients are determined by only demand side or supply side factors—the circumstances under which clear explanations would be valid. The problem is that because the coefficients of a hedonic function are the outcome of the interaction of consumer and producer optimizing conditions, it is not possible to interpret the function only in terms of, say, producer marginal costs or consumer marginal values. However, suppose the *production technology* τ was the same for each producing establishment. Buyers differ but sellers are identical. Then, instead of a confusing family of offer functions, there is a unique offer function with the hedonic function describing the prices of characteristics the firm will supply with the given ruling technology to the current mixture of tastes. The offer function becomes $p(z)$, since there is no distribution of τ to confuse it. There are different tastes on the consumer side, and so what appears in the market is the result of firms trying to satisfy consumer preferences all for a constant technology and profit level; the structure of supply is revealed by the hedonic price function. In Figure 21.1 only the expansion path traced out by, say S_H* akin to AA', would be revealed. Now, suppose sellers differ, but *buyers' tastes* α are identical. Here the family of *value functions* collapses to be revealed as the hedonic function $p(z)$ which identifies the structure of demand, such as AA' in Figure 21.1.[14] Diewert's (2003) approach follows a representative consumer, rather than consumers with different tastes, so

[14]Correspondingly, if the supply curves were perfectly inelastic, so that a change in price would not affect the supply of any of the differentiated products, then the variation in prices underlying the data and feeding the hedonic estimates would be determined by demand factors. The coefficients would provide estimates of user values. Similarly, if the supplying market were perfectly competitive, the estimates would be of resource costs. None of the price differences between differentiated items would be due to, say, novel configurations of characteristics, and no temporary monopoly profit would be achieved as a reward for this, or as a result of the

that the demand side alone can be identified. Triplett (1987, p. 632) notes that of these possibilities, uniformity of technologies is the most likely, especially when access to technology is unrestricted in the long run, while uniformity of tastes is unlikely. There may, of course, be segmented markets where tastes are more uniform to which specific sets of items are tailored and for which hedonic equations can be estimated for individual segments.[15] In some industries there may be a prior expectation of uniformity of tastes against uniformity of technologies and interpretation of coefficients will accordingly follow. In many cases, however, the interpretation may be more problematic. The pure producer approach requires assumptions of uniformity of technology and input prices which cannot of course be generally assumed. But the key assumption that will not generally be satisfied in the producer context is that each *producer is able to produce the entire array of hedonic models* whereas, in the consumer context, it is quite plausible that each consumer has the possibility of purchasing and consuming each model.

21.32 Third, issues relating to the estimation of the underlying supply and demand functions for characteristics have implications for the estimation of hedonic functions. In Appendix 21.2, identification and estimation issues will be considered in this light. Finally, the subsequent concern with new products in Section D of this chapter refers to demand functions. However, attention is now turned to hedonic *indices*. In the next section, these are noted to have a quite different application than that for the quality adjustment of noncomparable replacement items.

B.6 An alternative hedonic theoretical formulation

21.33 This section takes a consumer-based approach to deriving theoretical hedonic functions. It assumes:

- that every consumer has the same *separable sub-utility function*, $f(z_1,...,z_N)$ that gives the consumer the sub-utility $Z = f(z)$ from the purchase of one unit of the complex hedonic product that has the vector of characteristics $z \equiv (z_1,...,z_N)$[16];

- the sub-utility that the consumer gets from consuming Z units of the hedonic product is combined with the consumption of X units of a composite “other” product to give the consumer an overall utility of $u = U^t(X,Z)$ in period t, where U^t is the period t “macro” utility function. Rosen (1974; 38) normalized the price of X to be unity. This is not required in the present approach. Instead, there is an explicit period t price, p^t, for one unit of the general consumption product X.

exercise of market power. See Berndt (1983).

[15] Berry, Levinsohn, and Pakes (1995) provide a detailed and interesting example for automobiles in which makes are used as market segments, while Tauchen and Witte (2001) provide a systematic theoretical study of estimation issues for supply, demand, and hedonic functions where consumers and producers and their transactions are indexed across communities.

[16] It is not assumed that all possible models exist in the marketplace. In fact, we will assume that only a finite set of models exist in each period. It is assumed, however, that the consumer has preferences over all possible models, where each model is indexed by its vector of characteristics, $z = (z_1,...,z_N)$. Thus each consumer will prefer a potential model with characteristics vector $z^1 = (z_1^1,...,z_N^1)$ over another potential model with the characteristics vector $z^2 = (z_1^2,...,z_N^2)$ if and only if $f(z^1) > f(z^2)$.

The approach starts by considering the set of X and Z combinations that can yield the consumer's period t utility level, u^t . This is the set $\{(X,Z) : U^t(X,Z) = u^t\}$, which is the consumer's period t indifference curve over equivalent combinations of the general consumption product X and the hedonic product Z. The equation $U^t(X,Z) = u^t$ for X is solved as a function of u^t and Z; i.e.[17]

$$X = g^t(u^t ,Z) \tag{21.6}$$

It is assumed that the indifference curve slopes downward, and the stronger assumption is made that g^t is differentiable with respect to Z and

$$\partial g^t(u^t ,Z)/\partial Z < 0 \tag{21.7}$$

Let p^t and P^t be the prices for one unit of X and Z, respectively, in period t. The *consumer's period t expenditure minimization problem* may be defined as follows:

$$\min_{X,Z} \{p^tX + P^t Z : X = g^t(u^t ,Z)\} = \min_Z \{p^tg^t(u^t ,Z) + P^tZ\} \tag{21.8}$$

The first-order necessary condition for Z to solve equation (21.8) is:

$$p^t\partial g^t (u^t ,Z)/\partial Z + P^t = 0 \tag{21.9}$$

Equation (21.9) can be rearranged to give the price of the hedonic aggregate P^t as a function of the period t utility level u^t and the price of general consumption p^t :

$$P^t = -p^t\partial g^t(u^t ,Z)/\partial Z > 0 \tag{21.10}$$

where the inequality follows from assumption (21.7) above. The right-hand side of the equation (21.10) can now be interpreted as the consumer's *period t willingness to pay price function:*

$$w^t(Z,u^t, p^t) \equiv -p^t\partial g^t (u^t ,Z)/\partial Z \tag{21.11}$$

21.34 Thus, for each point (indexed by Z) on the consumer's period t indifference curve, equation (21.11) gives the amount of money the consumer would be willing to pay per unit of Z in order to stay on the same indifference curve, which is indexed by the utility level u^t . The *period t willingness to pay value function* v^t can now be defined as the product of the quantity of Z consumed times the corresponding per unit willingness to pay price, $w^t(Z,u^t,p^t)$:

$$v^t(Z,u^t, p^t) \equiv Z\, w^t(Z,u^t, p^t) = -Z\, p^t\partial g^t (u^t, Z)/\partial Z \tag{21.12}$$

where the last equality follows using equation (21.11). The function v^t is the counterpart to Rosen's (1974; 38) value or bid function; it gives us the amount of money the consumer is willing to pay in order to consume Z units. All of the above algebra has an interpretation that is independent of the hedonic model; it is simply an exposition of how to derive a willingness

[17] If the period t indifference curve intersects both axes, then $g^t(u^t ,Z)$ will only be defined for a range of non-negative Z up to an upper bound.

to pay price and value function using a consumer's preferences defined over two products.

21.35 It is assumed now that the consumer has a separable sub-utility function, $f(z_1,...,z_N)$ that gives the consumer the sub-utility $Z = f(z)$ from the purchase of one unit of the complex hedonic product[18] that has the vector of characteristics $z \equiv (z_1,...,z_N)$. Note that it has been assumed that the function f is time invariant. Let the consumer's period t utility function be $U^t(X, f(z))$. The above algebra on willingness to pay is still valid. In particular, the new period t willingness to pay price function, for a particular model with characteristics $z = (z_1,...,z_N)$, is:

$$w^t(f(z),u^t,p^t) \equiv -p^t\, \partial g^t(u^t,f(z))/\partial Z \qquad (21.13)$$

The new period t willingness to pay value function (which is the amount of money the consumer is willing to pay to have the services of a model with characteristics vector z) is:

$$v^t(f(z),u^t,p^t) \equiv f(z)w^t(f(z),u^t,p^t) = -f(z)\, p^t\, \partial g^t(u^t,f(z))/\partial Z \qquad (21.14)$$

21.36 Now suppose that there are K^t models available to the consumer in period t, where model k sells at the per unit price of P_k^t and has the vector of characteristics $z_k^t \equiv (z_{1k}^t,...,z_{Nk}^t)$ for $k = 1,2,...,K$. If the consumer purchases a unit of model k in period t, then the model price P_k^t can be equated to the appropriate willingness to pay value defined by equation (21.14), where z is replaced by z_k^t; i.e., the following equation should hold:

$$P_k^t = -f(z_k^t)\, p^t \partial g^t(u^t, f(z_k^t))/\partial Z \quad \text{for} \quad t = 1,...,T;\ \ k = 1,...,K^t \qquad (21.15)$$

What is the meaning of the separability assumption? Suppose the hedonic product is a car and suppose that there are only three characteristics: number of seats in the vehicle, fuel economy and horsepower. The separability assumption means that the consumer can trade off these three characteristics and determine the utility of any car with any mix of these three characteristics, independently of his or her other choices of products. In particular, the utility ranking of automobile models is independent of the number of children the consumer might have or what the price of petrol might be. Obviously, the separability assumption is not likely to be exactly satisfied in the real world, but this somewhat restrictive assumption is required to make our model tractable.

---

[18] If a consumer purchases, say, two units of a model at price P that has characteristics $z_1,...,z_N$ then we can model this situation by introducing an artificial model that sells at price $2P$ and has characteristics $2z_1,...,2z_N$. Thus the hedonic surface, $Z = f(z)$ consists of only the most efficient models including the artificial models. We do not assume that $f(z)$ is a quasi-concave or concave function of z. In normal consumer demand theory, $f(z)$ can be assumed to be quasi-concave without loss of generality because linear budget constraints and the assumption of perfect divisibility will imply that "effective" indifference curves enclose convex sets. As Rosen (1974; 37-38) points out, however, in the case of hedonic products, the various characteristics cannot be untied. Moreover, perfect divisibility cannot be assumed and not all possible combinations of characteristics will be available on the marketplace. Thus the usual assumptions made in "normal" consumer demand theory are not satisfied in the hedonic context. Note also that while we place a smoothness assumption on the macro functions $g^t(u,Z)$, namely the existence of the partial derivative $\partial g^t(u,Z)/\partial Z$, we do not place any smoothness restrictions on the hedonic sub-utility function $f(z)$.

Another aspect of our model needs some further explanation. It is being explicitly assumed that consumers cannot purchase fractional units of each model.; they can purchase only a non-negative integer amount of each model. That is, indivisibilities are being explicitly assumed on the supply side of our model. Thus, in each period, there are only a finite number of models of the hedonic product available. While the consumer is assumed to have continuous preferences over all possible combinations of characteristics $(z_1,...,z_N)$ in each period, there are only a finite number of isolated models that are available on the market.

At this point, the model is further specialized. It is assumed that every consumer has the same hedonic sub-utility function[19] $f(z)$ and consumer i has the following *linear indifference curve macro utility function* in period t:

$$g_i^t(u_i^t,Z) \equiv -a^t Z + b_i^t u_i^t \quad \text{for} \quad t = 1,...,T \ \text{ and } \ i = 1,...,I \qquad (21.16)$$

where a^t and b_i^t are positive constants.

For each period t and each consumer i, the period t indifference curve between combinations of X and Z is linear, with the constant slope $-a^t$ being the same for all consumers.[20] Note that this slope is allowed to change over time. Now differentiate equation (21.16) with respect to Z and substitute this partial derivative into equation (21.15). The resulting equation is:[21]

$$P_k^t = p^t a^t f(z_k^t) \quad \text{for} \quad t = 1,...,T \ \text{ and } \ k = 1,...,K^t \qquad (21.17)$$

Define the aggregate price of one unit of Z in period t as:[22]

$$r_t \equiv p^t a^t \quad \text{for} \quad t = 1,...,T \qquad (21.18)$$

[19] The sameness assumption is very strong and needs some justification. This assumption is entirely analogous to the assumption that consumers have the same homothetic preferences over, say, food. Although this assumption is not justified for some purposes, it suffices for the purpose of constructing a price index for food, since we are mostly interested in capturing the substitution effects in the aggregate price of food as the relative prices of food components vary. In a similar fashion, we are interested in determining how the "average" consumer values a faster computer speed against more memory; i.e., we are primarily interested in hedonic substitution effects.

[20] We do not require a linear indifference curve globally but only locally over a certain range of purchases. Alternatively, we can view the linear indifference curve as providing a first-order approximation to a non-linear indifference curve.

[21] Comparing equation (21.17) with equation (21.15), it can be seen that the simplifying assumptions (21.16) enable us to get rid of the terms $\partial g^t(u_i^t, f(z_k^t))/\partial Z$, which depend on individual consumer indifference curves between the hedonic commodity and other products. If we had individual household data on the consumption of hedonic and other products, then we could use normal consumer demand techniques in order to estimate the parameters that characterized these indifference curves.

[22] There has been a switch to subscripts from superscripts in keeping with the conventions for parameters in regression models; i.e., the constants r_t will be regression parameters in what follows. Note also that r_t is the product of the price of the "other" product p^t times the period t slope parameter a^t . We need to allow this slope parameter to change over time in order to be able to model the demand for high technology hedonic products, which have been falling in price relative to "other" products; i.e., we think of a^t as decreasing over time for high technology products.

Now substitute equation (21.18) into equation (21.17) in order to obtain our *basic system of hedonic equations*.[23]

$$P_k^t = r_t\, f(z_k^t) \quad \text{for} \quad t = 1,\ldots,T \quad \text{and} \quad k = 1,\ldots,K^t \tag{21.19}$$

21.37 All that is needed is to postulate a functional form for the hedonic sub-utility function f and add a stochastic specification to equation (21.19) to yield a basic hedonic regression model. The unknown parameters in f along with the period t hedonic price parameters r_t can then be estimated.[24] It is possible to generalize the above model, but get the same model (21.19) if the composite “other” product X is replaced by $h(x)$, where x is a consumption vector and h is a linearly homogeneous, increasing and concave aggregator function. Instead of equation (21.17), under these new assumptions, the following equation results:

$$P_k^t = c(p^t)a^t f(z_k^t) \qquad \text{for } t = 1,\ldots,T \quad \text{and } k = 1,\ldots,K^t \tag{21.20}$$

where p^t is now the vector of prices for the x products in period t and c is the unit cost or expenditure function that is dual to h.[25] Now redefine r_t as $c(p^t)a^t$ and the basic system of hedonic equations (21.19) is still obtained. Equation (21.19) has one property that is likely to be present in more complex and realistic models of consumer choice. This property is that the model prices in period t are homogeneous of degree one in the general price level p^t . Thus if p^t is replaced by λp^t for any $\lambda > 0$ (think of a sudden hyperinflation where λ is large), then equations (21.17) and (21.19) imply that the model prices should become λP_k^t . Note that this homogeneity property will not hold for the following additive hedonic model:

$$P_k^t = r_t + f(z_k^t) \quad \text{for} \quad t = 1,\ldots,T \quad \text{and} \quad k = 1,\ldots,K^t \tag{21.21}$$

21.38 Thus hedonic regressions based on the linear model (21.21) may be ruled out on a priori grounds. Note that hedonic models that take the logarithm of the model price P_k^t as the dependent variable will tend to be consistent with basic hedonic equations (21.19) whereas

---

[23] The basic model ends up being very similar to one of Muellbauer’s (1974; 988-989) hedonic models; see in particular his equation (32).

[24] It is possible to rework the above theory and give it a producer theory interpretation. The counterpart to the expenditure minimization problem (21.8) is now the following profit maximization problem: $\max_{x,z}$ $\{P^tZ - w^tX : X = g^t(k^t, Z)\}$ where Z is hedonic output and P^t is a period t price for one unit of the hedonic output, w^t is the period t price of a variable input and X is the quantity used of it, k^t is the period t quantity of a fixed factor (capital say) and g^t is the firm’s factor requirements function. Assuming that $Z = f(z)$, we end up with the following producer theory counterpart to equation (21.15): $P_k^t = f(z_k^t)\ \partial g^t(k^t, f(z_k^t))/\partial Z$. The counterpart to assumption (21.16) is, for firm i, $g_i^t(k_i^t, Z) \equiv a^tZ - b_i^t\, k_i^t$ and the counterpart to equation (21.17) becomes $P_k^t = w^t a^t f(z_k^t)$. The producer theory model assumptions are, however, not as plausible as the corresponding consumer theory model assumptions. In particular, it is not very likely that each producer will have the same period t aggregate price for a unit of variable input w^t and it is not very likely that each firm producing in the hedonic market will have the same technology parameter a^t . The key assumption that will not generally be satisfied in the producer context is that each producer is able to produce the entire array of hedonic models; whereas, in the consumer context, it is quite plausible that each consumer has the possibility of purchasing and consuming each model.

[25] Define c as $c(p^t) \equiv \min_x \{p^t x : h(x) = 1\}$ where $p^t x$ denotes the inner product between the vectors p^t and x.

linear models like (21.21) will not be consistent with the normal linear homogeneity properties implied by microeconomic theory.

B.7 Markups and imperfect competition

21.39 In Section B.5 it was shown there was some ambiguity in the interpretation of hedonic coefficients. A user-value or resource-cost interpretation was possible if there was uniformity in buyer's tastes or suppliers' technologies, respectively. In Section B.6 an assumption of price-taking behavior on the part of firms was introduced and a formal setting given to a user value interpretation, albeit involving some restrictive assumptions. Yet the approaches in Sections B.5 and B.6 both assume perfectly competitive behavior, and the discussion extends now to the effects of markups in imperfect competition. Feenstra (1995) notes that in imperfect competition, when pricing is above marginal cost, the hedonic function should include a term for the price-cost markup.

21.40 Pakes (2001) has developed the argument focusing on the study of new products as the result of prior investments in product development and marketing. A competitive marginal cost-pricing assumption would require that either (i) products with identical characteristics are developed from such investments, so that the law of one price for these identical products will eliminate any margin, or (ii) all products lose their investment (markup) in the new products. Neither of these is reasonable. Indeed, varying markups are a feature of differentiated products (see Feenstra and Levinsohn, 1995, for example). Pakes (2001) argued that markups should change over time. When new products are introduced, the improvements, and associated markups, are directed to characteristics where markups have previously been high. The markups on existing products with these characteristics will fall, and hedonic coefficients will thus, change over time. Pakes (2001) also argued that there may be an ambiguity as to the signs of the coefficients—that there is no economic reason to expect a positive relationship between price and a desirable characteristic. Such a conclusion would be at odds with a resource-cost or user-value approach. If the characteristics being compared are *vertical*—that is, they are characteristics, of which everyone would like more—then we can expect the sign to be positive. However, Pakes (2001) has argued that the sign on *horizontal* characteristics—that is, for which the ordering of the desirable amounts of characteristics is not the same for all consumers—can be negative. The entry of new products aimed at some segments of the market may drive down the markup on products with more desirable attributes. For example, some consumers may have a preference for television sets with smaller screen sizes and be willing to pay a premium price. Indeed, the required technology for the production of these sets may have required increased investment and thus, increased expected markups. It may be that the quality of the picture on these sets is such that it drives down the price of large-sized sets, resulting in an inverse relationship between price and screen size, where the latter is taken as one variable over the full range of screen sizes. Prior (to the modeling) information on the two markets would allow the regression equation to be appropriately specified, with dummy slope and intercepts for the ranges of screen sizes with new and old technologies.

21.41 Pakes (2001) takes the view that no meaning can be attributed to estimated coefficients and predicted values should be used for price comparisons of models of different quality attributes, rather than the individual coefficients. There are many good reasons for this, as discussed in Chapter 7, Section E.4.3 and Section G.2.2, and the Appendix 21.1 to this chapter. Yet, it must be stressed that for vertical characteristics the coefficients may be quite meaningful, and even for horizontal characteristics or new characteristics, embodied with the latest research and development, some sense can be made by recourse to the above considerations. But again, theory does not support any easy answer to the interpretation of the coefficients from hedonic regressions. Their relevance is that they emanate from market data, from the often complex interaction of demand and supply and strategic pricing decisions. That theory warns us not to give simplistic interpretations to such coefficients, and allows an understanding of the factors underlying them, is a strength of theory. Yet they remain and are generally regarded (Shultze and Mackie, 2002) as the most promising objective basis for estimating the marginal value of quality dimensions of products, even though a purist interpretation is beyond their capability. [26]

C. Hedonic Indices

C.1 The need for such indices

21.42 In Section A it was noted that hedonic functions are required for two purposes with regard to a quality adjustment. The first is when an item is no longer produced and the replacement item, whose price is used to continue the series, is of a different quality from the original price basis. The differences in quality can be established in terms of different values of a subset of the *z* price-determining variables. The coefficients from the hedonic regressions, as estimates of the monetary value of additional units of each quality component *z*, can then be used to adjust the price of the old item so that it is comparable with the price of the new[27]—so that, again, like is compared with like. This process could be described as "patching," in that an adjustment is needed to the price of the old (or new replacement) series for the quality differences, to enable the new series to be patched onto the old. A second use of hedonic functions referred to in Section A is for estimating *hedonic indices.* These are suitable when the pace and scale of replacements of items is substantial and an extensive use of patching might (i) lead to extensive errors if there were some error or bias in the quality-adjustment process and (ii) lead to sampling from a biased replacement universe as outlined in Section A. Hedonic indices use data in each period from a sample of items that should include those with substantial share of sales revenue—sampling in each period from the double universe. There is no need to establish a price basis and for respondents to keep quoting prices from that basis. What is required are samples of items to be redrawn in each month along with information on their prices, characteristics z_i, and, possibly, quantities or values. The identification of multiple characteristics in the hedonic regressions controls for quality differences, as opposed to the matching of price quotes on the same price basis by the

[26]Diewert (2002f) goes further in suggesting positive sign restrictions should be imposed on the coefficients in the econometric estimation, particularly when the hedonic regression is being used to adjust the price of a replacement item in order to make it comparable with the price of an item that has disappeared.

[27]Mechanisms for such adjustments are varied, as outlined in Chapter 7, Section E.4.3, and Triplett (2004). They include using the coefficients from the salient set of characteristics or using the predicted values from the regression as a whole and, in either case, making the adjustment to the old for comparison with the new, or to the new for comparison with the old, or some effective average of the two.

respondents. A number of procedures for estimating hedonic indices are briefly considered below.

C.2 Theoretical characteristics price indices

21.43 Theoretical cost of living indices are defined in Chapter 17 and practical index number formulae are considered as estimates of these indices. Theoretical cost of living index numbers are defined here not just on the goods produced, but also on their characteristics. The Konüs (1924) family of *true cost of living indices* pertaining to two periods, where the consumer faces the strictly positive price vectors $p^0 \equiv (p_1^0,\ldots,p_N^0)$ and $p^1 \equiv (p_1^1,\ldots,p_N^1)$ in periods 0 and 1 respectively, was defined in Chapter 17 as the ratio of the minimum costs of achieving the same utility level $u \equiv f(q)$ where $q \equiv (q_1,\ldots,q_N)$ is a positive reference quantity vector; i.e.,

$$P_K(p^0,p^1,q) \equiv C[u,p^1]/C[u,p^0] = C[f(q),p^1]/C[f(q),p^0] \tag{21.22}$$

For theoretical indices in characteristic space, the revenue functions are also defined over goods made up of bundles of characteristics represented by the hedonic function:[28]

$$P_K(p^0,p^1,q) \equiv C[u,p^1,p(z_1)]/C[u,p^0,p(z_0)] = C[f(q),p^1,p(z_1)]/C[f(q),p^0,p(z_0)] \tag{21.23}$$

21.44 The theoretical price index defined by equation (21.23) is a ratio of the period 1 to period 0 hypothetical costs to consumers of achieving a given utility. Equation (21.23) incorporates substitution effects: if the prices of some characteristics increase more than others, then utility-maximizing consumers can switch their output mix of characteristics in favour of such characteristics. The numerator in equation (21.23) is the cost of the maximum utility that the consumer could attain if faced with the product prices and implicit hedonic shadow prices of period 1, $p^1$and $p(z^1)$, while the denominator in equation (21.23) is the maximum utility that the consumer could attain if faced with the product and characteristic prices of period 0, p^0 and $p(z^0)$. Note that all of the variables in the numerator and denominator functions are exactly the same, except that the product price and characteristic price vectors differ. This is a defining characteristic of a price index. As with the economic indices in Chapter 15, there is of course an entire family of indices depending on which reference utility level is chosen. Some explicit formulations are considered in paragraphs 21.48 to 21.58, including a base period 0 reference level and a current period 1 reference level analogous to the derivation of the Laspeyres and Paasche indices in Chapter 17, paragraph 17.12. Before considering such hedonic indices in paragraphs 21.48 to 21.58, two simpler formulations are first considered: hedonic regressions using dummy variables on time, and hedonic imputation indices. They are simple, and widely used because they require no information on quantities or weights. They also do not require matched data, so can be

[28] Triplett (1987) and Diewert (2002d), following Pollak (1975), consider a two-stage budgeting process whereby that portion of utility concerned with items defined as characteristics has its theoretical index defined in terms of a cost-minimizing selection of characteristics, conditioned on an optimum output level for composite and hedonic commodities. These quantities are then fed back into the second-stage maximization of overall revenue.

used when resampling all of the data. Yet their interpretation from economic theory is therefore more limited on account of this. As will be shown in the Appendix, however, weighted formulations are possible using a weighted least squares estimator.

C.3 Hedonic regressions and dummy variables on time

21.45 Let there be K characteristics of a product, and let model or item i of the product in period t have the vector of characteristics $z_i^t \equiv [z_{i1}^t,...,z_{iK}^t]$ for $i = 1,....,N$ and $t = 1,...,T$. Denote the price of model i in period t by p_i^t. A hedonic regression of the price of model i in period t on its characteristics set z_i^t is given by

$$\ln p_i^t = \gamma_1 + \gamma^t D^t + \sum_{k=1}^{K} \beta_k z_{ik}^t + \varepsilon_i^t \qquad (21.24)$$

where D_t are dummy variables for the time periods, D_2 being 1 in period $t = 2$, zero otherwise; D_3 being 1 in period $t = 3$, zero otherwise, and so on. The coefficients γ_t are estimates of quality-adjusted price changes, having controlled for the effects of variation in quality (via $\sum_{k=1}^{K} \beta_k z_{ik}^t$) — although see Goldberger (1968) and Teekens and Koerts (1972) for the adjustment for estimation bias.

21.46 The above approach uses the dummy variables on time to compare prices in period 1 with prices in each subsequent period. In doing so, the γ parameters are constrained to be constant over the period $t = 1,\ldots,T$. Such an approach is fine retrospectively, but in real time the index may be estimated as a fixed-base or chained-base formulation. The *fixed-base* formulation would estimate the index for period 1 and 2, $I_{1,2}$, using equation (21.24) for $t = 1, 2$; the index for period 3, $I_{1,3}$, would use equation (21.24) for t = 1, 3; for period 4, $I_{1,4}$, using equation (21.24) for $t = 1, 4$; and so forth. In each case the index constrains the parameters to be the same over the current and base period. A fixed-base, bilateral comparison using equation (21.24) makes use of the constrained parameter estimates over the two periods of the price comparison. A *chained* formulation would estimate $I_{1,4}$, for example, as the product of a series of links: $I_{1,4} = I_{1,2} \times I_{2,3} \times I_{3,4}$.[29] Each successive binary comparison or link is combined by successive multiplication. The index for each link is estimated using equation (21.24). Because the periods of time being compared are close, it is generally more likely that the constraining of parameters required by chained-time dummy hedonic indices is considered to be less severe than that required of their fixed base counterparts.

21.47 There is no explicit weighting in these formulations, and this is a serious disadvantage. In practice, cut-off sampling might be employed to include only the most important items. If sales data are available, a WLS (weighted by relative sales shares—see Appendix 21.1 and Diewert (2005)) estimator should be used instead of an ordinary least squares (OLS) estimator.[30] A WLS estimator is equivalent to replicating the sample in proportion to the weights and applying an OLS estimator.

---

[29] Chapter 15, Section F contains a detailed account of chained indices.

[30] Ioannidis and Silver (1999) and Bode and van Dalen (2001) compared the results from these different estimators, finding notable differences, but not in all cases (see also Silver and Heravi, 2002).

C.4 Period-on-period hedonic indices

21.48 An alternative approach to comparing prices in period 0 and 1 is to estimate a hedonic regression for period 1 and insert the values of the characteristics of each model existing in period 0 into the period 1 regression to predict, for each item, its price $\hat{p}_i^t\left(z_i^1\right)$. This would generate predictions of the price of items existing in period 0, at period 1 shadow prices, $\hat{p}_i^t\left(z_i^1\right)$, $i = 1,...,N$. These prices (or an average) can be compared with (the average of) the actual prices of models $i = 1,\ldots N$ models in period 0. The averages may be arithmetic, as in a Dutot index, or geometric, as in a Jevons index. The arithmetic formulation is defined as follows:

$$\frac{\sum_{i=1}^{N}(1/N)\hat{p}_i^1(z_i^0)}{\sum_{i=1}^{N}(1/N)p_i^0(z_i^0)} \qquad (21.25a)$$

21.49 Alternatively, the characteristics of models existing in period 1 can be inserted into a regression for period 0. Predicted prices of period 1 items generated at period 0 shadow prices (or an average) can be compared with (the average of) the actual prices in period *t*:

$$\frac{\sum_{i=1}^{N}(1/N)p_i^1(z_i^1)}{\sum_{i=1}^{N}(1/N)\hat{p}_i^0(z_i^1)} . \qquad 21.25b)$$

21.50 For a fixed-base bilateral comparison using either equation (21.25a) or (21.25b), the hedonic equation need be estimated only for one period. The denominator in equation (21.25a) is the average observed price in period 0, which should be equal to the average price a hedonic regression based on period 0 data will predict using period 0 characteristics. The numerator, however, requires an estimated hedonic regression to predict period 0 characteristics at period 1 hedonic prices. Similarly, in equation (21.25b), a hedonic regression is required only for the denominator. For reasons analogous to those explained in Chapters 15, 16, and 17, a symmetric average of these indices should have some theoretical support.

21.51 Note that all the indices described in Sections C.1 and C.2 use all the data available in each period. If there is a new item, for example, in period 4, it is included in the data set and its quality differences controlled for by the regression. Similarly, if old items drop out, they are still included in the indices in the periods in which they exist. This is part of the natural estimation procedure, unlike using matched data and hedonic adjustments on non-comparable replacements when items are no longer produced.

21.52 As with the dummy variable approach, there is no need for matched data . Yet there is also no explicit weighting in these formulations and this is a serious disadvantage. Were data on quantities or values available, it is immediately apparent that such weights could be attached to the individual $i = 1,\ldots N$ prices or their estimates. This is considered in the next section.

C.5 Superlative and exact hedonic indices

21.53 In Chapter 17 Laspeyres and Paasche bounds were defined on a theoretical basis, as were superlative indices, which treat both periods symmetrically. These superlative formulae included the Fisher index, which was seen in Chapter 16 to have desirable axiomatic properties. Furthermore, the Fisher index was supported from economic theory as a symmetric average of the Laspeyres and Paasche bounds, and was found to be the most suitable such average of the two on axiomatic grounds. The Törnqvist index also possessed desirable axiomatic properties, seemed to be best from the stochastic viewpoint, and also did not require strong assumptions for its derivation from the economic approach as a superlative index. The Laspeyres and Paasche indices were found to correspond to (be exact for) underlying (Leontief) aggregator functions with no substitution possibilities, while superlative indices were exact for flexible functional forms including the quadratic and translog forms for the Fisher and Törnqvist indices respectively. If data on prices, characteristics and quantities are available, then analogous approaches and findings arise for hedonic indices; see Fixler and Zieschang (1992) and Feenstra (1995). Exact bounds on such an index were defined by Feenstra (1995). Consider the theoretical index in equation (21.23), but now only defined over items in terms of their characteristics. The prices are still of items, but they are wholly defined through $p(z)$. An arithmetic aggregation for a linear hedonic equation finds that a Laspeyres upper bound (as quantities supplied *decrease* with increasing relative prices) is given by:

$$\frac{\sum_{i=1}^{N} x_i^{t-1}\hat{p}_i^t}{\sum_{i=1}^{N} x_i^{t-1}p_i^{t-1}} = \sum_{i=1}^{N} s_i^{t-1}\left(\frac{\hat{p}_i^t}{p_i^{t-1}}\right) \geq \frac{C\left(u^{t-1}, p\left(z^t\right)\right)}{C\left(u^{t-1}p\left(z^{t-1}\right)\right)} \tag{21.26a}$$

where the right-hand side expression is the ratio of the cost of achieving a period *t*-1 level of utility (u^{t-1}) during periods (t-1) and t, where utility is a function of the vector of quantities; i.e., $u^{t-1} = f\left(x^{t-1}\right)$; the price comparison is evaluated at a fixed level of period *t*-1 quantities and s_i^{t-1} are the shares in the total value of expenditure on product *i* in period *t*-1:
$s_i^{t-1} = x_i^{t-1}p_i^{t-1} \Big/ \sum_{j=1}^{N} x_j^{t-1}p_j^{t-1}$

21.54 The difference between a Laspeyres formula and the left-hand side of equation (21.26a) is that the price in the numerator of the left-hand side of equation (21.26a) is a predicted price:

$$\hat{p}_i^t \equiv \hat{p}_i^t\left(z_i^{t-1}\right) = \sum_{k=1}^{K} \beta_k^t z_{ik}^{t-1} \tag{21.26b}$$

or, if a non-comparable replacement is used, then the predicted price adjusts for the difference in quality between the old and new items. That is, the predicted price

$$\hat{p}_i^t \equiv p_i^t - \sum_{k=1}^{K} \beta_k^t (z_{ik}^t - z_{ik}^{t-1}) \tag{21.26c}$$

is the price in period t adjusted for the sum of the changes in each quality characteristic weighted by their coefficients derived from a linear hedonic regression. Note that the summation is over the same i in both periods since replacements are included when an item is missing, and (21.26c) adjusts the prices in period t for quality differences via $\sum_{k=1}^{K} \beta_k^t (z_{ik}^t - z_{ik}^{t-1})$.

21.55 A Paasche lower bound is estimated as:

$$\frac{\sum_{i=1}^{N} x_i^t p_i^t}{\sum_{i=1}^{N} x_i^t \hat{p}_i^{t-1}} = \left[\sum_{j=1}^{N} s_j^t \left(\frac{\hat{p}_j^{t-1}}{p_j^t} \right) \right]^{-1} \leq \frac{C\left(u^t, p\left(z^t\right)\right)}{C\left(u^t, p\left(z^{t-1}\right)\right)} \tag{21.27a}$$

where $s_i^t = x_i^t p_i^t \Big/ \sum_{j=1}^{N} x_j^t p_j^t$ and

$$\hat{p}_i^{t-1} \equiv \sum_{k=1}^{K} \beta_k^{t-1} z_{ik}^t \tag{21.27b}$$

$$\hat{p}_i^{t-1} \equiv p_i^{t-1} + \sum_{k=1}^{K} \beta_k^{t-1} (z_{ik}^t - z_{ik}^{t-1}) \tag{21.27c}$$

which are the imputation and replacement adjustments, respectively. The latter are the prices in periods t-1 adjusted for the sum of the changes in each quality characteristic weighted by their respective coefficients derived from a linear hedonic regression.

Following from the inequalities in (17.5) where the Laspeyres P_L and Paasche P_P indices form bounds (17.8) on their "true" P_K economic theoretic indices:

$$P_L \leq P_K \leq P_P \ or \ P_P \leq P_K \leq P_L. \tag{21.28}$$

a suitable index is thus a Fisher geometric mean of the Laspeyres P_L and Paasche P_P indices, which incorporate hedonic adjustments for quality differences.

21.56 The approach based on using superlative and exact hedonic indices thus, first, applies the coefficients from hedonic regressions to changes in the characteristics to adjust observed prices for quality changes. Second, it incorporates a weighting system using data on the quantities sold of each model and their characteristics, rather than treating each model as equally important. Finally, it has a direct correspondence to the formulation defined using economic theory.

21.57 Semi-logarithmic hedonic regressions would supply a set of β coefficients suitable for use with the base and current period geometric bounds:

$$\prod_{i=1}^{N}\left(\frac{p_i^t}{\hat{p}_i^{t-1}}\right)^{s_i^t} \le \frac{C\left(u^t, p\left(z^t\right)\right)}{C\left(u^t, p\left(z^{t-1}\right)\right)} \le \prod_{i=1}^{N}\left(\frac{\hat{p}_i^t}{p_i^{t-1}}\right)^{s_i^{t-1}} \tag{21.29a}$$

$$\hat{p}_i^{t-1} \equiv \exp\left[\sum_{k=1}^{K}\beta_k^{t-1} z_{ik}^t\right]$$

$$\hat{p}_i^t \equiv \exp\left[\sum_{k=1}^{K}\beta_k^t z_{ik}^{t-1}\right] \tag{21.29b}$$

$$\hat{p}_i^{t-1} \equiv p_i^{t-1} \exp\left[\sum_{k=1}^{K}\beta_k^{t-1}(z_{ik}^t - z_{ik}^{t-1})\right]$$

$$\hat{p}_i^t \equiv p_i^t \exp\left[-\sum_{k=1}^{K}\beta_k^t(z_{ik}^t - z_{ik}^{t-1})\right] \tag{21.29c}$$

21.58 In the inequality (21.29a), the two bounds on the respective theoretical indices have been shown to be brought together. The calculation of such indices is relatively straightforward for matched data, but for unmatched data is no small task. For an example of its application for unmatched comparisons over time, see Silver and Heravi (2002 and 2003) and Chapter 7, paragraphs 7.132 to 7.152, and see Kokoski, Moulton and Zieschang (1999) for matched price comparisons across regions of a country.

21.59 Exact hedonic indices can also be defined using the theoretical framework outlined by Diewert (2003a).[31] Recall the basic hedonic equation (21.19). Assume that the price P_k^t is the average price for all the models of type k sold in period t and let q_k^t be the number of units sold of model k in period t. Recall that the number of models in the market-place during period t is K^t . Assume that there are K models in the market-place over all T periods in our sample period. If a particular model k is not sold at all during period t, then it will be assumed that P_k^t and q_k^t are both zero. With these conventions in mind, the *total value of consumer purchases during period t* is equal to:

$$\sum_{k=1}^{K} P_k^t q_k^t = \sum_{k=1}^{K} r_t f\left(z_k\right) q_k^t \qquad \text{for} \qquad t = 1,\ldots,T \tag{21.30}$$

21.60 The hedonic sub-utility function f has done all of the hard work in the model in converting the utility yielded by model k in period t into a "standard" utility $f(z_k)$ that is cardinally comparable across models. For each model type k, it is only necessary to multiply by the total number of units sold in period t, q_k^t , in order to obtain the *total period t market quantity of the hedonic product*, Q_t say. This yields[32]:

---

[31] The assumptions are quite different from those made by Fixler and Zieschang (1992) who took yet another approach to the construction of exact hedonic indices.

[32] This is a counterpart to the quantity index defined by Muellbauer (1974; 988) in one of his hedonic models; see his equation (30). Of course, treating r_t as a price for the hedonic commodity quantity aggregate defined by equation (21.31) can be justified by appealing to Hicks' (1946; 312-313) aggregation theorem, since the model

$$Q_t \equiv \sum_{k=1}^{K} f(z_k) q_k^t \qquad \text{for} \qquad t = 1,...,T \qquad (21.31)$$

21.61 The aggregate price for the hedonic product corresponding to Q_t is r_t. Thus in the highly simplified model outlined in Section B.6, the *aggregate exact period t price and quantity* for the hedonic product are r_t and Q_t defined by equation (21.31), which can readily be calculated provided the parameters in the hedonic regression have been estimated and provided that data on quantities sold are available each period q_k^t .[33] Once r_t and Q_t have been determined for $t = 1,...,T$, then these aggregate price and quantity estimates for the hedonic product can be combined with the aggregate prices and quantities of non-hedonic products using normal index number theory. Any of the index number formulae considered in Chapter 17, including Laspeyres, Paasche and Fisher, can be accordingly defined based on the use of quantity information.

21.62 The above illustrates how weighted quality-adjusted price index number formulae might be constructed using data on prices, quantities and characteristics of an item. The method using dummy variables of time, described in Section C.3, does not require matched data. Appendix 21.1 discusses a weighting system. The use of weighted superlative indices for matched data is outlined above. Weighted superlative indices may also be applied to unmatched data, using a method outlined in Chapter 7 and in Silver and Heravi (2001a) (2001b) and (2003). But what of unweighted indices, which was the concern of the initial section of this chapter? What correspondence does the unweighted hedonic dummy time index (outlined in Section C.3), which uses all of the data, have to the matched unweighted index number formulae? This is a critical question for product areas where there is a rapid turnover of items. It was suggested above that the dummy time variable method be used instead of the matched method. So how do they differ for unweighted indices? The effect and use of weights is considered in the Appendix to this chapter.

C6. The difference between the period on period and time dummy approaches

21.63 The dummy variable method outlined in section C3 and the period-on-period hedonic indexes, outlined in sections C4 and C5—also referred to as “hedonic imputation indexes” by Silver and Heravi (2006b) and as “characteristic price index numbers” by Triplett (2004) —not only correct price changes for changes in the quality of items purchased, but also allow the indexes to incorporate matched and unmatched models. They provide a means by which price changes can be measured in product markets where there is a rapid turnover of differentiated models. However, they can yield quite different results. Silver and Heravi (2006b) provides a formal exposition of the factors underlying such differences and the implications for choice of method. This was undertaken for the Törnqvist

---

prices $p_k^t = r_t f(z_k)$ all have the common factor of proportionality r_t.

[33] If data are available for the q_k^t , then it is best to run sales-weighted regressions, as discussed in Appendix 21.1. If we do not have complete market data on individual model sales but we do have total sales in each period, then the hedonic regression model can be run using a sample of model prices, and period t sales can be divided by our estimated r_t parameter in order to obtain an estimator for Q_t.

index, but the analysis can be readily extended to other formulas. They found that differences between the two approaches may arise from both parameter instability over the two periods compared and changes over the two periods compared in the characteristics of the models sold, and that such differences are compounded when both such changes occur. They further showed that similarities between the two approaches resulted if there was little difference in either component change.

21.64 The above in section C has illustrated how weighted index number formulas might be constructed using data on prices, quantities, and characteristics for an item when the data are not matched. But for analytical purposes it is useful to decompose price changes into that due to matched price changes, that due to unmatched new models introduced, and that due to unmatched old models that are retired. The analysis is useful for determining the bias in just using matched models.

C.7 Decomposing price changes into matched and unmatched components

21.65 Triplett (2004) argues and Diewert (2003) shows formally that an unweighted geometric mean Jevons index for matched data gives the same result as a logarithmic hedonic index run on the same data. There is simply no point in estimating hedonic indices using *matched* data. An index from a dummy variable hedonic regression such as (21.29), but in log-log form, for matched models can be shown (Aizcorbe, Corrado, and Doms, 2001 and Silver and Heravi, 2005b) to equal:

$$\ln p_t/p_{t-1} = \sum_{m\in M_t} (\ln p_{mt} - Z_m)/M_t \ - \sum_{m\in M_{t-1}} (\ln p_{mt-1} - Z_m)/M_{t-1}\,, \qquad (21.32)$$

where m is the matched sample and Z_t and Z_{t-1} are in principle the quality adjustments to the dummy variables for time in equation (21.24), that is, $\sum_{k=1}^{K}\gamma_k z_{tk}$. Equation (21.32) is simply the difference between two geometric means of quality-adjusted prices. The sample space m = $M_t = M_{t-1}$ is the same model in each period. Consider the introduction of a new model n introduced in period t with no counterpart in t – 1 and the demise of an old model o so it has no counterpart in t. So in period t, M_t is composed of the period t matched items m and the new items n, and in period t – 1, M_{t-1} is composed of the period t-1 matched items m and the old items. Silver and Heravi (2005b) have shown the dummy variable hedonic comparison to now be

$$\begin{aligned}
\ln p_t/p_{t-1} &= [m/(m+n)\sum_m (\ln p_{mt} - Z_m)/m \ + \ n/(m+n)\sum_n (\ln p_{nt} - Z_n)/n] \\
&\quad - [m/(m+o)\sum_m (\ln p_{mt-1} - Z_m)/m + o/(m+o)\sum_o (\ln p_{ot-1} - Z_o)/o] \\
&= [m/(m+n)\sum_m (\ln p_{mt} - Z_m)/m \ - \ m/(m+o)\sum_m (\ln p_{mt-1} - Z_m)/m] \\
&\quad + [n/(m+n)\sum_n (\ln p_{nt} - Z_n)/n \ - o/(m+o)\sum_o (\ln p_{ot-1} - Z_o)/o]. \qquad (21.33)
\end{aligned}$$

21.66 Consider the second expression in equation (21.33). First there is the change for m

matched observations. This is the change in mean prices of matched models *m* in period *t* and *t*–1 adjusted for quality. Note that the weight in period *t* for this matched component is the proportion of matched observations to all observations in period *t*. And, similarly, for period *t*–1, the matched weight depends on how many unmatched old observations are in the sample. In the last line of equation (21.33) the change is between the unmatched new and the unmatched old mean (quality adjusted) prices in periods *t* and *t*–1. Thus, matched methods can be seen to ignore the last line in equation (21.33) and will thus differ from the hedonic dummy variable approach. The hedonic dummy variable approach in its inclusion of unmatched old and new observations can be seen from equation (21.33) possibly to differ from a geometric mean of matched prices changes. The extent of any difference depends, in this unweighted formulation, on the proportions of old and new items leaving and entering the sample and on the price changes of old and new items relative to those of matched items. If the market for products is one in which old quality-adjusted prices are unusually low while new quality-adjusted prices are unusually high, then the matched index will understate price changes (see Silver and Heravi, 2005b, and Berndt, Ling, and Kyle, 2003, for examples). Different market behavior will lead to different forms of bias. The above expression is for unweighted price changes, but the principles extend to similar findings for weighted price changes and, by association, weighted index numbers. As shown in Silver and Heravi (2005b). As noted in the Appendix to this chapter, and argued in Diewert (2005), different weighting systems in a weighted least squares hedonic regression correspond to different index number formulae.

D. New Goods and Services

21.67 This section briefly highlights theoretical issues relating to the incorporation of new goods into the index. Practical issues were outlined in Chapter 8, paragraphs 8.43 to 8.60. The term “new goods” will be used here to refer to those that provide a substantial and substantive change in what is provided, as opposed to more of a currently available set of service flows, such as a new model of an automobile that has a bigger engine. In this latter instance, there is a continuation of a service and production flow and this may be linked to the service flow and production technology of the existing model. The practical concern with the definition of new goods as against quality changes is that the former cannot be easily linked to existing items as a continuation of an existing resource base and service flow, because of the very nature of their “newness”. There are alternative definitions; Oi (1997) directs the problem of defining “new” goods to that of defining a monopoly. If there is no close substitute, the good is new. A monopoly supplier may be able to supply an item with new combinations of the hedonic *z* characteristics because of a new technology and have a monopoly power in doing so, but in practice the new good can be linked via the hedonic characteristics set to the existing goods. In this practical sense, such goods are not considered “new” for the purposes of the Manual.

21.68 The terminology adopted here is that used by Merkel (2000) for the measurement of producer price indices, but considered in the context of consumer price indices (CPIs). The aim is to distinguish between *evolutionary* and *revolutionary* goods. Evolutionary goods are replacement or supplementary models which continue to provide a similar service flow, perhaps in new ways or to different degrees. In contrast, revolutionary goods are goods that are substantially different from pre-existing goods. They are generally produced on entirely new production lines or with substantially new production inputs and processes than those used to produce preexisting goods. These differences make it virtually impossible, both from a theoretical and practical standpoint, to quality adjust between a revolutionary good and any preexisting good.

21.69 The main concern regarding the incorporation of new goods into the CPI is the decision on the need and timing for their inclusion. Waiting for a new good to be established or waiting for the rebasing of an index before incorporating new products may lead to errors in the measurement of price changes if the unusual price movements at critical stages in the product life cycles are ignored. There are practical approaches to the early adoption of both evolutionary and revolutionary goods. These are outlined in Chapter 8 Section D.3. For evolutionary goods, such strategies include the rebasing of the index, re-sampling of items and introduction of new goods as directed sample *substitutions*; see Merkel (2000). Also of use are hedonic quality adjustments and indices outlined in Chapter 7, Section E4, and Section C above that facilitate the incorporation of such evolutionary goods, since they possess a similar characteristic set to existing goods, but deliver different quantities of these characteristics. The modified-short-run or chained framework outlined in Chapter 7 Sections H-G may also be more appropriate for product areas with a high turnover of items. These approaches can incorporate the price change of new goods into the index as soon as prices are available for two successive periods, although issues relating to the proper weighting of such changes may remain.

21.70 However, for revolutionary goods, however, substitution may not be appropriate. First, revolutionary goods may not be able to be defined within the existing classification systems. Second, they may be primarily sold in a new outlet, which will require extending the sample to such outlets. Third, there will be no previous items to match them against and to make a quality adjustment to prices, since by definition they are substantially different from pre-existing goods. And, finally, there is no weight to attach to the new outlet or item. Sample *augmentation* is appropriate for revolutionary goods, as opposed to sample substitution for evolutionary goods. It is necessary to bring the new revolutionary goods into the sample in addition to what exists. This may involve extending the classification, the sample of outlets, and the item list within new or existing outlets (Merkel, 2000).

21.71 The second measurement issue with respect to new products is the incorporation of the welfare effect of those products at introduction. The preceding discussion has been concerned with the incorporation of price changes into the index once two successive quotations are available. Yet there is a gain to the consumer when comparing the price in the first of these periods with the price in the period that preceded its introduction *had it existed.* In the context of the CPI, the appropriate period 1 shadow price for the new good is that price that just induces the consumer of the new good to consume zero quantities in the preceding period. This is a hypothetical price. If it is relatively high in the period before the introduction of the good, but the actual price in the period of introduction is much lower, then the introduction of the new good is clearly of some benefit to the consumer. To ignore this benefit, and the change from the virtual price to the actual price in its period of introduction, is to ignore something of the price movements that give rise to expenditure changes.

21.72 The sample augmentation procedures miss the effects on price between the period preceding the introduction of a new good and its introduction. There exist in economic theory and practice the tools for estimating such effects; see Hicks (1940) and Diewert (1980; 498-503). This involves setting a virtual price in the period before introduction. This price is the one at which supply is set to zero. The virtual price is compared with the actual price in the period of introduction and this is used to estimate the welfare gain from the introduction of the good. Hausman (1997) provides some estimates of consumer welfare for the introduction of a new brand of breakfast cereals, Apple-Cinnamon Cheerios. He concludes:

> "The correct economic approach to the evaluation of new goods has been known for over fifty years, since Hick's pioneering contribution. However, it has not been implemented by government statistical agencies, perhaps because of its complications and data requirements. Data are now available. The impact of new goods on consumer welfare appears to be significant according to the demand estimates of this paper, the CPI for cereal may be too high by about 25 percent because it does not account for new cereal brands. An estimate this large seems worth worrying about."

21.73 Shapiro and Wilcox (1997; 144) share the same concerns:

> "……the rare new item that delivers services radically different from anything previously available. For example, even the earliest generation of personal computers allowed consumers to undertake tasks that previously would have been prohibitively expensive. "This problem can be solved only by estimating the consumer surplus created by the introduction of each new item. Hausman (1997) argues that this must involve explicit modeling of the demand for each new item. ……..Although explicit modeling of demand may be of dubious practicality for widespread implementation in the CPI, strategic application in a few selected cases might be worthwhile."

21.74 The expertise required for such estimates is considerable, and even when applied, is not beyond dispute; see Bresnahan (1997) on this last point. An alternative approach is outlined for the CPI by Balk (2000b) with empirical estimates provided by de Haan (2001), the details being provided in Chapter 8 and Appendix 8.2. While this approach is simpler than that undertaken by Hausman (1997), both require considerable statistical and econometric expertise. The inclusion of such effects on a routine basis is not something being actively considered, even by statistical offices with well-developed systems.[34]

[34] Even if virtual prices were estimated, there would still be problems with including new goods in indices such as the Laspeyres index because of the absence of weights in the base period.

Appendix 21.1: Some Econometric Issues

21.75 Hedonic regression estimates will have been seen in Chapter 7 to have potential use for the quality adjustment of prices. There are a number of issues arise from the specification and estimation of hedonic regressions, the use of diagnostic statistics, and courses of action when the standard OLS assumptions are seen to break down. Many of these issues are standard econometric ones and not the subject of this *Manual*. This is not to say they are unimportant. The use of hedonic regressions will require some econometric or statistical expertise, but suitable texts are generally available. See Berndt (1991)—particularly the chapter on hedonic regressions—and Maddala (1988) and Kennedy (2003), among many others. Modern statistical and econometric software have adequate diagnostic tests for testing when OLS assumptions break down. There remain, however, some specific issues that merit attention, although it must be stressed that these points are over and above, and should not be taken to diminish, the important standard econometric issues found in econometric texts.

Identification and appropriate estimators

21.76 Wooldridge (1996, pp. 400–01) has shown on standard econometric grounds that the estimation of supply and demand functions by OLS is biased *and this bias carries over to the estimation of the hedonic function*. It is first useful to consider estimation issues in the supply and demand functions. These functions are rarely estimated in practice. The more common approach is to estimate offer functions, with the marginal price offered by the firm dependent upon chosen attributes (product characteristics) and firm characteristics, and to estimate *bid* or value functions, with the marginal prices paid by a consumer dependent on chosen attributes and consumer characteristics.[35] As noted earlier, the observed prices and quantities are the result of the interaction of structural demand and supply equations and the distributions of producer technologies and consumer tastes; they cannot reveal the parameters of these offer and value functions. Rosen (1974, pp. 50–51) suggested a procedure for determining these parameters. Since these estimates are conditioned on tastes (α) and technologies (τ), the estimation procedure needs to include empirical measures or "proxy variables" of α and τ. For the tastes α of consumers, the empirical counterparts may be sociodemographic and economic variables, which may include age, income, prices and quantities of nonhedonic products demanded by households,[36] education, and geographical region. For technologies τ, variables may include technologies and factor prices. First, the hedonic equation is estimated without these variables in the normal manner using the best-fitting functional form. This is to represent the price function consumers and producers face when making their decisions. Then, an implicit marginal price function is computed for each characteristic as $\partial p(z)/\partial z_i = \hat{p}_i(z)$ where $\hat{p}(z)$ is the estimated hedonic equation. Bear in mind that in normal demand and supply studies for *products*, the prices are observed in the

[35] These are equivalent to inverse demand (supply) functions, with the prices dependent upon the quantities demanded (supplied) and the individual consumer (producer) characteristics.

[36] The consumer theory approach used by Diewert (2003) to deriving the hedonic function rested on rather strong separability assumptions on consumer preferences. Once these separability assumptions are relaxed, the demand for nonhedonic commodities will provide a means for identification of the hedonic preferences.

market. For *characteristics* that are unobserved, the first stage must be to estimate the parameters from the hedonic regression. The actual values of each z_i bought and sold is then inserted into each implicit marginal price function to yield a numerical value for each characteristic. These marginal values are used in the second stage[37] of estimation as endogenous variables for the estimation of the demand side:

(A21.1) $\hat{p}_i(z) = F(z_1,\ldots,z_K, \alpha^*)$

where α* are the proxy variables for tastes.

The supply side estimating equations might look like:

(A21.2) $\hat{p}_i(z) = F(z_1,\ldots,z_K, \tau^*)$,

where τ^* are the proxy variables for technologies.

The variables τ^* drop out when there is no variation in technologies and $\hat{p}_i(z)$ is an estimate of the offer function. Similarly the variables α* drop out when sellers differ and buyers are identical and cross-section estimates trace out compensated demand functions.

21.77 Epple (1987) has argued that Rosen's modeling strategy is likely to give rise to inappropriate estimation procedures of the demand and supply parameters. The hedonic approach to estimating the demand for characteristics has a difficulty arising from the fact that marginal prices are likely to be endogenous—they depend on the amount of each characteristic consumed and must be estimated from the hedonic function rather than observed directly. There are two resulting problems. First, there is an identification problem (see Epple, 1987) because both the marginal price of a characteristic and the inverse bid depend on the levels of characteristics consumed. Second, if important characteristics are unmeasured and they are correlated with measured characteristics, the coefficients on measured characteristics will be biased. This applies to all econometric models, but it is particularly relevant to hedonic models; on this point see Wooldridge (1996, 400–01). The equilibrium conditions for characteristic prices imply functional relationships among the characteristics of demanders, suppliers, and products. This in turn reduces the likelihood that important excluded variables will be uncorrelated with the included variables of the model (see also Bartik, 1988, on this point). The bias arises because buyers are differentiated by characteristics (y,α) and sellers by technologies τ. The type of item buyers will purchase is related to (y,α) and the type sellers provide to τ. On the plane of combinations of z transacted, the equilibrium ones chosen may be systematically related; the characteristics of buyers are related to those of sellers. Epple (1987) uses the example of stereo equipment: the higher income of some buyers leads to purchases of high-quality equipment and the technical competence of sellers leads them to provide it. The consumer and producer characteristics may be correlated.

21.78 Wooldridge (1996, pp. 400–01) suggests that individual consumer and firm

[37]This two-stage approach is common in the literature, though Wooldridge (1996) discusses the joint estimation of the hedonic and demand and supply side functions as a system.

characteristics such as income, education, and input prices should be used as instruments in estimating hedonic functions. In addition, variables other than a good's characteristics should be included as instruments if they are price determining, such as geographical location—say proximity to ports, good road systems, climate, and so on. Communities of economic agents are assumed, within which consumers consume and producers produce for each other at prices that vary across communities for identical goods. Variables on the characteristics of the communities will not in themselves enter the demand and supply equation but are price determining for observed prices recorded across communities. Tauchen and Witte (2001) provide a systematic investigation of the conditions under which consumer and producer and community characteristics will affect the hedonic parameter estimates for a single-regression equation estimated across all communities. A key concern is whether the hedonic price function error term represents factors that are unobserved by both the economic agents and the researcher, or by the researcher only. In the latter case the error term may be correlated with the product attributes and instrumental variable estimation is required. If the error term is *not* correlated with the product characteristics—preferences are quasi-linear—then a properly specified hedonic regression, including community-specific characteristics or appropriate slope dummies, can be estimated using OLS. In other cases, depending on the correlation between consume and producer characteristics, assumptions about the error term and the method of incorporating community characteristics into the regression, instrumental variables, including consumer or producer or community dummy or characteristics, may need to be used.

Functional form

21.79 Triplett (1987; 2004) argues that neither classical utility theory nor production theory can specify the functional form of the hedonic function.[38] This point dates back to Rosen (1974, p. 54) who describes the observations as being "..a joint-envelope function and cannot by themselves identify the structure of consumer preferences and producer technologies that generate them." A priori judgments about what the form should look like may be based on ideas about how consumers and production technologies respond to price changes. These judgments are difficult to make when the observations are jointly determined by demand and supply factors but not impossible in rare instances. However, it is complicated when pricing is with a markup, the extent of which may vary over the life cycle of a product. Some tied combinations of characteristics will have higher markups than others. New item introductions are likely to be attracted to these areas of characteristic space, and this will have the effect of increasing supply and thus, lowering the markup and price (Cockburn and Anis, 1998; Feenstra, 1995, p. 647; and Triplett, 1987, p. 38). This again must be taken into account in any a priori reasoning—not an easy or straightforward matter.

21.80 It may be that in some cases the hedonic function's functional form will be very straightforward. For example, prices on the websites for options for products are often additive. The underlying cost and utility structure are unlikely to jointly generate such linear functions, but the producer or consumer is also paying for the convenience of selling in this

[38]Arguea, Hsiao, and Taylor (1994) propose a linear form on the basis of arbitrage for characteristics, held to be likely in competitive markets, although Triplett (2004) argues that this is unlikely to be a realistic scenario in most commodity markets.

way and is willing to bear losses or make gains if the cost or utility at higher values of z are priced lower/worth more than the price set. But, in general, the data should convey what the functional form should look like, and imposing artificial structures simply leads to specification bias. For examples of econometric testing of hedonic functional form, see Cassel and Mendelsohn (1985); Cropper, Deck, and McConnell (1988),\; Rasmussen and Zuehlke (1990); Bode and van Dalen (2001); and Curry, Morgan, and Silver (2001).

21.81 The three forms prevalent in the literature are linear, semilogarithmic, and double-logarithmic (log-log). A number of studies have used econometric tests, in the absence of a clear theoretical statement, to choose between them. There have been a large number of hedonic studies, and, as illustrated in Curry, Morgan, and Silver (2001), in many of these the quite simple forms do well, at least in terms of the $\bar{R}^2$ presented, and the parameters accord with a priori reasoning, usually on the consumer side. Of the three popular forms some are favored in testing. For example, Murray and Sarantis (1999) favored the semilogarithmic form, while in others—for example Hoffmann (1998)—the three functional forms were found to scarcely differ in terms of their explanatory power. That the parameters from these simple forms accord with a priori reasoning, usually from the consumer side, is promising, but researchers should be aware that such matters are not assured. Of the three forms, the semilogarithmic form has much to commend it. The interpretation of its coefficients is quite straightforward—the coefficients represent proportionate changes in prices arising from a unit change in the value of the characteristic.[39] This is a useful formulation since quality adjustments are usually undertaken by making multiplicative instead of additive adjustments (see Chapter 7, Section C.3). The semilogarithmic form, unlike the log-log model, can also incorporate dummy variables for characteristics that are either present, $z_i = 1$, or not, $z_i = 0$.[40]

21.82 More complicated forms are possible. Simple forms have the virtue of parsimony and allow more efficient estimates to be made for a given sample. However, parsimony is not something to be achieved at the cost of misspecification bias. First, if the hedonic function is estimated across multiple independent markets, then interaction terms are required (see Mendelsohn, 1984, for fishing sites). Excluding them is tantamount to omitting variables and inappropriately constraining the estimated coefficients of the regression. Tauchen and Witte (2001) have outlined the particular biases that can arise from such omitted variables in hedonic studies. Second, it may be argued that the functional form should correspond to the aggregator for the index—linear for a Laspeyres index, logarithmic for a geometric Laspeyres index, translog for a Törnqvist index, and quadratic for a Fisher index (Chapter

[39]It is noted that the anti-log of the OLS-estimated coefficients is not unbiased—the estimation of semilogarithmic functions as transformed linear regressions requires an adjustment to provide minimum-variance unbiased estimates of parameters of the conditional mean. A standard adjustment is to add one-half of the coefficient's squared standard error to the estimated coefficient (Goldberger, 1968, and Teekens and Koerts, 1972).

[40]Diewert (2002f) argues against the linear form on the grounds that, while the hedonic model is linear, the estimation required is of a nonlinear *regression* model, and the semi-log and log-log models are linear *regression* models. He also notes that semi-log form has the disadvantage against the log-log of not being able to impose constraints of constant returns to scale. Diewert (2002d) I think that you call this Diewert (2003) elsewhere; i.e., my NBER paper that started out as a comment on your paper. also argues for the use of nonparametric functional forms and the estimation of linear generalized dummy variable hedonic regression models. This has been take up in Curry, Morgan, and Silver (2001), who use neural networks that are shown to work well, although the variable set required for their estimation has to be relatively small.

17). However, as Triplett (2004) notes, the purpose of estimating hedonic regressions is to adjust prices for quality differences, and imposing a functional form on the data that is inconsistent with the data might create an error in the quality adjustment procedure. Yet, as Diewert (2003) notes, flexible functional forms encompass these simple forms. The log-log form is a special case of the translog form as in equation 17.11, and the semi-log form is a special case of the semi-log quadratic form as in equation 17.16. If there are a priori reasons to expect interaction terms for specific characteristics, as illustrated in the example in Chapter 7, Section E.4, then these more general forms allow this, and the theory of hedonic functions neither dictates the hedonic form nor restricts it.

Changing tastes and technologies

21.83 The estimates of the coefficients may change over time. Some of this will be attributed to sampling error, especially if multicollinearity is present, as discussed below. But, in other cases, it may be a genuine reflection of changes in tastes and technologies. If a subset of the estimated coefficients from a hedonic regression is to be used to quality adjust a noncomparable replacement price, then the use of estimated out-of-date coefficients from some previous period to adjust the prices of the new replacement model would be inappropriate. There would be a need to update the indices as regularly as the changes demand.[41] For estimating hedonic indices, the matter is more complicated. The coefficients in a simple dummy time-period model as in Section C.3 now have different estimates of the parameters in each period. Silver (1999), using a simple example, shows how the estimate of quality adjusted price change from such a dummy-variable model requires a reference basket of characteristics. This is apparent for the hedonic imputation indices where separate indices using base-and current-period characteristics are estimated. A symmetric average of such indices is considered appropriate. A hedonic index based on a time dummy variable implicitly constrains the estimated coefficients from the base and current periods to be the same. Diewert (2003) formalizes the problem of choosing the reference characteristics when comparing prices over time when the parameters of the hedonic function may themselves be changing over time. He finds the results of hedonic indices to *not* be invariant to the choice of reference-period characteristic vector set *z*. The use of a sales (quantity) weighted average vector of characteristics proposed by Silver (1999) is considered, but Diewert notes that over long time periods this may become unrepresentative.[42] Of course, if the dummy-variable approach is used in a chained formulation as outlined in Section C.3, the weighted averages of characteristics remain reasonably up to date, though chaining has its own pros and cons (see Chapter 15). A fixed-base alternative noted by Diewert (2003) is to use a Laspeyres-type comparison with the base-period parameter set, and a Paasche-type current-period index with the current-period parameter set, and take the geometric mean of the two indices for reasons similar to those given in Chapter 17, Section B.3. The resulting Fisher-type index is similar to that given in equation (21.32) proposed by Feenstra (1995).[43] A feature of the time dummy approach in is that it implicitly takes a symmetric average of the coefficients by constraining

[41]In Chapter 15, Section C.3, the issue of adjusting the base versus the current period's price is discussed, since there are different data demands.

[42]Other averages may be proposed—for example, the needs of an index representative of the "typical" establishment would be better met by a trimmed mean or median.

[43]Diewert (2002c) also suggests matching items where possible and using hedonic regressions to impute the prices of the missing old and new ones. Different forms of weighting systems, including superlative ones, can

them to be the same. But what if, as is more likely the case, only base-period hedonic regression coefficients are available? Since hedonic indices based on a symmetric average of the coefficients are desirable, the spread or difference between estimates based on either a current or a reference-period characteristics set is an indication of potential bias, and estimates of such spread may be undertaken retrospectively. If the spread is large, estimates based on the use of a single period's characteristics set, say the current period, should be treated with caution. More regular updating of the hedonic regressions is likely to reduce spread because the periods being compared will be closer and the characteristics of the items in the periods compared more similar.

Weighting

21.84 OLS estimators implicitly treat each item as being of equal importance, although some items will have quite substantial sales, while for others sales will be minimal. It is axiomatic that an item with sales of more than 5,000 in a month should not be given the same influence in the regression estimator as one with a few transactions. Products with very low sales may be at the end of their life cycles or be custom made. Either way, their (quality-adjusted) prices and price changes may be unusual.[44] Such observations with unusual prices should not be allowed to unduly influence the index.[45] The estimation of hedonic regression equations by a WLS estimator is preferable. This estimator minimizes the sum of *weighted* squared deviations between the actual prices and the predicted prices from the regression equation, as opposed to OLS estimation, which uses an equal weight for each observation. There is a question as to whether to use quantity (volume) or expenditure weights. The use of quantity weights can be supported by considering the nature of their equivalent "price." Such prices are the average (usually the same) price over a number of transactions. The underlying sampling unit is the individual transaction, so there is a sense that the data may be replicated as being composed of, say, 12 individual observations using an OLS estimator, as opposed to a single observation with a weight of 12 using a WLS estimator. Both would yield the same result. Inefficient estimates arise if the variance of the errors, $V(u_i)$, is not constant—that is, they are heteroskedastic. WLS is equivalent to assuming that the error variances are related to the weights in a multiplicative manner, say $V(u_i) = \sigma^2 w_i^2$.[46] A priori notions as to whether a hedonic regression model predicts better/worse at different levels of quantities or expenditures may help in identifying which weights are appropriate; however, statistical tests or plots of heteroskedasticity may be more useful.

21.85 The sole use of statistical criteria for deciding on which weighting system to use has rightfully come under some criticism. Diewert (2002c and 2005) and Silver (2002) have argued that what matters is whether the estimates are representative of the target index in

---

be applied to this set of price data in each period for both matched and unmatched data.

[44]Such observations have higher variances of their error terms, leading to imprecise parameter estimates. This would argue for the use of WLS estimators with quantity sold as the weight. This is one of the standard treatments for heteroskedastic errors (see Berndt, 1991).

[45]See Berndt, Ling, and Kyle (2003), Cockburn and Anis (1998), and Silver and Heravi (2002) for examples. Silver and Heravi (2002) show old items have above-average leverage effects and below-average residuals. Not only are they different, but they exert undue influence for their size (number of observations).

[46]Estimating an equation for which each variable is divided by the square root of the weight using OLS is an equivalent procedure.

mind. Conventional target index numbers such as Laspeyres, Paasche, Fisher, and Törnqvist weight price changes by expenditure shares, and the latter two formulas have received support from the axiomatic, stochastic, fixed-base, and economic theoretic approaches, as shown in Chapters 15–18. Thus, value weights are preferred to quantity weights: "The problem with quantity weighting is this: it will tend to give too little weight to cheap models that have low amounts of useful characteristics" (Diewert, 2002c, p. 8). He continues to argue that for a WLS estimator of hedonic time dummy variable indices, expenditure *share* weights should be used, as opposed to the *value* of expenditure, to avoid inflation increasing period 1 value weights, resulting in possible heteroskedastic residuals. Furthermore, for a semilogarithmic hedonic function when models are present in both periods, the average expenditure shares in periods 0 and 1 for *m* items, $½(s_{m0} + s_{m1})$, should be used as weights in the WLS estimator. If only matched models exist in the data, then such an estimator may be equivalent to the Törnqvist index. If an observation *m* is only available in one of the periods, its weight should be s_{m0} or s_{m1} accordingly, and the WLS estimator provides a *generalization* of the Törnqvist index.

21.86 Silver (2002) has shown that a WLS estimator using value weights will not necessarily give each observation a weight equal to its relative value. The estimator will give more weight to those observations with high leverage effects and residuals. Observations with values of characteristics with large deviations from their means—say, very old or new models—have relatively high leverage. New and old models are likely to be priced at quite different prices than those predicted from the hedonic regression, even after taking into account their different characteristics. Such prices result, for example, from a pricing strategy designed to skim segments of the market willing to pay a premium for a new model, or from a strategy to charge relatively low prices for an old model to dump it to make way for a new one. In such cases the influence these models have on deriving the estimated coefficients will be over and above that attributable to their value weights. Silver (2002) suggests that leverage effects should be calculated for each observation, and those with high leverage and low weights should be deleted, and the regression re-run. Thus, while quantity or value weights are preferable to no weights (that is, OLS), value weights are more appropriate than quantity ones and, even so, account should be taken of observations with undue influence.

21.87 Diewert (2002f) has also considered the issue of weighting with respect to the time dummy hedonic indices outlined in Section C.6. The use of WLS by value involves weights being applied to observations in both periods. However, if, for example, there is high inflation, then the sales values for a model in the current period will generally be larger than those of the corresponding model in the base period, and the assumption of homoskedastic residuals is unlikely to be met. Diewert (2002f and 2005) suggests the use of expenditure *shares* in each period, as opposed to values, as weights for WLS for time dummy hedonic indices. He also suggests that an average of expenditure shares in the periods being compared be used for matched models.

21.88 Data on sales are not always available for weights, but the major selling items can generally be identified. In such cases, it is important to restrict the number of observations of items with relatively low sales, the extent of the restriction depending on the number of observations and the skewness of the sales distribution. In some cases, items with few sales provide the variability necessary for efficient estimates of the regression equation. In other cases, their low sales may be due to factors that make them unrepresentative of the hedonic surface, their residuals being unusually high. An example is low-selling models about to be dumped to make way for new models. Unweighted regressions may thus suffer from a sampling problem—even if the prices are perfectly quality adjusted, the index can be biased because it is unduly influenced by low-selling items with unrepresentative price-characteristic relationships. In the absence of weights, regression diagnostics have a role to play in helping to determine whether the undue variance in some observations belongs to such unusual low-selling items.[47]

21.89 There is a situation in which an unweighted OLS estimator is preferred. This is when markets are in perfect hedonic equilibrium. Observations with unusual characteristics, say old or new models, would take values which were particularly dispersed from their means and thus increase the variation of the sample for the same underlying model. Such increased variation leads to an increase in the efficiency of the estimates. However, theory and empirical observation (see Silver and Heravi, 2005b) find that such outliers do not have the same structural relationships as other models. If the sales shares of these new and old models are low relative to the number of models they represent in the market, then an OLS regression would give them undue weight.

21.90 Multicollinearity.

21.91 There are a priori reasons to expect for some products that the variation in the values of one characteristic will not be independent of one or a linear combination of other *z* characteristics. As a result, parameter estimates will be unbiased, yet imprecise. To illustrate this, a plot of the confidence interval for one parameter estimate against another collinear one is often described as elliptical, since the combinations of possible values they may take can easily drift from, say, high values of β_1 and low β_2 to higher values of β_2 and lower of $\beta_{1.}$ Since the sample size for the estimates is effectively reduced, relatively small additions to and deletions from the sample may affect the parameter estimates more than would be expected. These are standard statistical issues, and the reader is referred to Maddala (1988) and Kennedy (2003). In a hedonic regression, multicollinearity might be expected as some characteristics may be technologically tied to others. Producers including one characteristic may need to include others for it all to work, while for the consumer side, purchasers buying, for example, an up-market brand may expect a certain bundle of features to come with it. Triplett (2004) argues strongly for the researcher to be aware of the features of the product and consumer market. There are standard, though not completely reliable, indicators of multicollinearity (such as variance inflation factors), but an exploration of its nature is greatly aided by an understanding of the market along with exploration of the effects of

[47]A less formal procedure is to take the standardized residuals from the regression and plot them against model characteristics that may denote low sales, such as certain brands (makes) or vintage (if not directly incorporated) or some technical feature that makes it unlikely that the item is being bought in quantity. Higher variances may be apparent from the scatter plot. If certain features are expected to have, on average, low sales, but seem to have high variances, leverages, and residuals (see Silver and Heravi, 2002), a case exists for at least downplaying their influence. Bode and van Dalen (2001) use formal statistical criteria to decide between different weighting systems and compare the results of OLS and WLS, finding, as with Ioannidis and Silver (1999), that different results can arise.

including and excluding individual variables on the signs and coefficients and on other diagnostic test statistics (see Maddala, 1988).[48]

21.92 If a subset of the estimated coefficients from a hedonic regression is to be used to quality adjust a noncomparable replacement price, and if there is multicollinearity *between* variables in this subset *and* other independent variables, then the estimates of the coefficients to be used for the adjustment will be imprecise. The multicollinearity effectively reduces the sample size, and some of the effects of the variables in the subset may be wrongly ascribed to the other independent variables. The extent of this error will be determined by the strength of the multiple-correlation coefficient between all such “independent” variables (the multicollinearity), the standard error or “fit” of the regression, the dispersion of the independent variable concerned, and the sample size. These all affect the precision of the estimates, since they are components in the standard error of the *t*-statistics. Even if multicollinearity is expected to be quite high, large sample sizes and a well-fitting model may reduce the standard errors on the *t*-statistics to acceptable levels. If multicollinearity is expected to be severe, the predicted value for an item’s price may be computed using the whole regression and an adjustment made using the predicted value, as explained in Chapter 7, Section E.4, since there is a sense in which it would not matter whether the variation was wrongly attributed to either β_1 or β_2. If dummy variable hedonic *indices* are being calculated (Section B.3 above), the time trend will be collinear with an included variable if a new feature appears in a new month for the vast majority of the items, so that the data are not rich enough to allow the separate effects of the coefficient on the time dummy to be precisely identified. The extent of the imprecision of the coefficient on the time dummy will be determined by the aforementioned factors. A similar argument holds for omitted variable bias.

Omitted variable bias

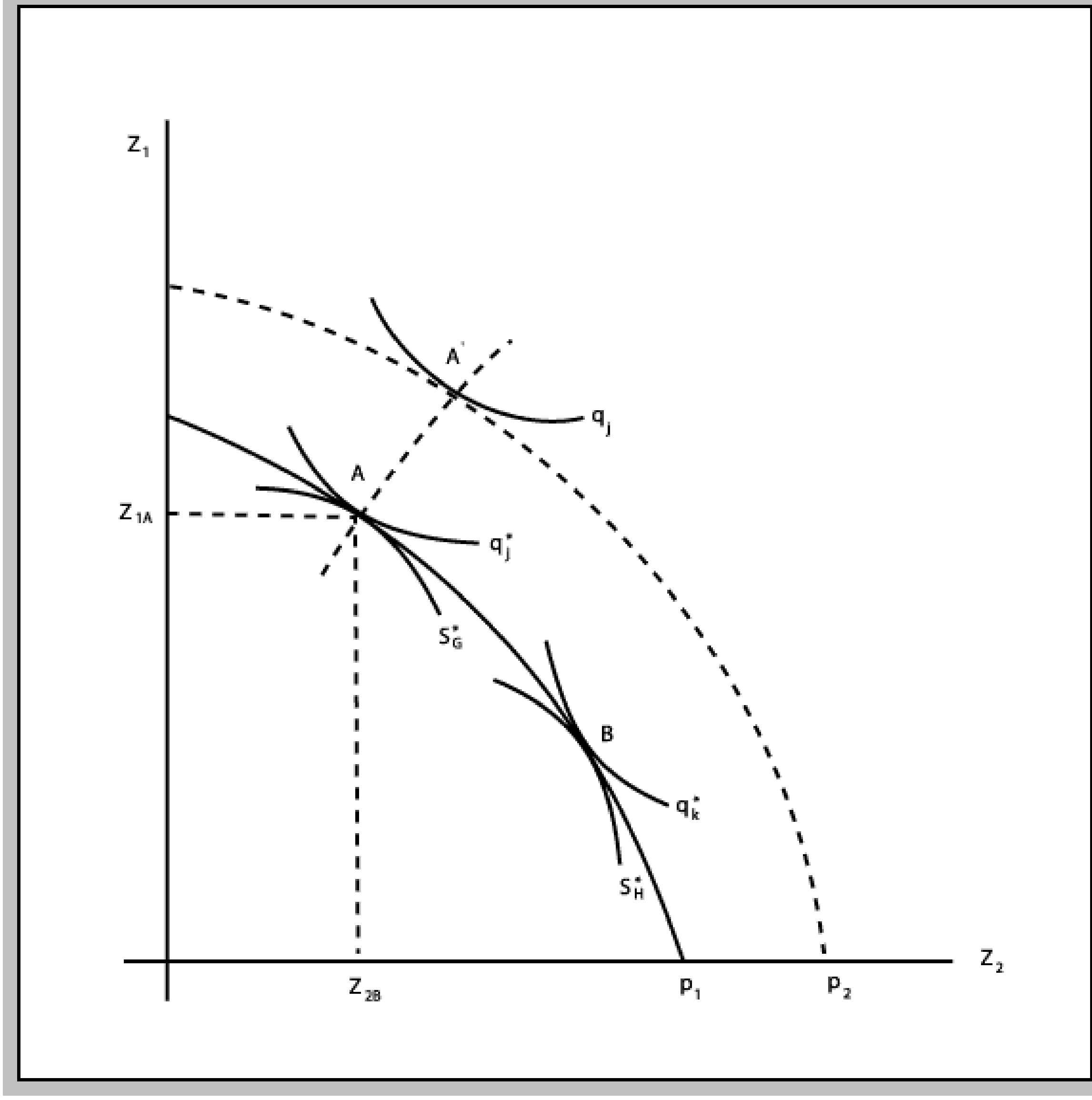
21.93 The exclusion of tastes and technology and community characteristics has already been discussed. The concern here is with product characteristics. Consider again the use of a subset of the estimated coefficients from a hedonic regression to quality adjust a noncomparable replacement price. It is well established that multicollinearity of omitted variables with included variables leads to bias in the estimates of the coefficients of included ones. If omitted variables are *independent* of the included variables, then the estimates of the coefficients on the included variables are unbiased. This is acceptable in this instance; the only caveat is that it may be that the quality adjustment for the replacement item also requires an adjustment for these omitted variables, and this, as noted by Triplett (2004), has to be undertaken using a separate method and data. But what if the omitted variable is multicollinear with a subset of included ones, and these included ones are to be used to quality adjust a non-comparable item? In this case, the coefficients on the subset of the included variables may be wrongly picking up some of the omitted variables’ effects. The coefficients will be used to quality adjust prices for items that differ only with regard to this subset of included variables, and the price comparison will be biased if the characteristics of both included and omitted variables have different price changes. For hedonic *indices* using a dummy time trend, the estimates of quality-adjusted price changes will suffer from a similar

[48]Triplett (2004) stresses the point that $\bar{R}^2$ alone is insufficient for this purpose.

bias if excluded from the regression are omitted variables multicollinear with the time change. What are picked up as quality-adjusted price changes over time may, in part, be changes due to the prices of these excluded variables. This requires that the prices on the omitted characteristics follow a different trend. Such effects are most likely when there are gradual improvements in the quality of items, such as the reliability and safety of consumer durables,[49] which are difficult to measure, at least for the sample of items in real time. The quality–adjusted price changes will thus overstate price changes in such instances.

[49]There are some commodity areas, such as airline comfort, that have been argued to have overall patterns of decreasing quality.

Figure 21.1. Consumption and Production Decisions for Combinations of Characteristics



Appendix 21.1 Some econometric issues

1. Hedonic regression estimates are seen in Chapter 7 to have a potential use for quality adjustment to prices. A number of issues arise from the specification and estimation of hedonic regressions, the use of diagnostic statistics, and courses of action when the standard assumptions of ordinary least squares (OLS) are seen to break down. Many of these issues are standard econometric ones and not the subject of this manual. This is not to say they are unimportant. The use of hedonic regressions requires some econometric and statistical expertise, but suitable texts are generally available; see Berndt (1991) – particularly the chapter on hedonic regressions – and Maddala (1988) and Kennedy (1998) amongst many others. Modern statistical and econometric software has adequate diagnostic tests for testing when OLS assumptions break down. There remain, however, some specific aspects which merit attention; these points are over and above the important standard econometric considerations dealt with in econometric texts.

Identification and appropriate estimators

2. Wooldridge (1996, pp. 400-401) has shown on standard econometric grounds that the estimation of supply and demand functions by OLS is biased and this bias carries over to the estimation of the hedonic function. It is first useful to consider estimation issues regarding demand and supply functions. The demand and supply functions are rarely estimated in practice. The more common approach is to estimate *offer* functions, with the marginal price offered by the firm dependent upon chosen attributes (product characteristics) and firm characteristics, and *bid* or value functions, with the marginal prices paid by a consumer dependent upon chosen attributes and consumer characteristics.[50] As noted earlier, the observed prices and quantities are the result of the interaction between structural demand and supply equations and the distributions of producer technologies and consumer tastes; they cannot reveal the parameters of the offer and value functions. Rosen (1974, pp. 50-51) suggested a procedure for determining these parameters. Since these estimates are conditioned on tastes (α) and technologies (τ), the estimation procedure needs to include empirical measures or "proxy variables" of α and τ. For the tastes α of consumers, the empirical counterparts may be socio-demographic and economic variables, which may include age, income, education and geographical region. For technologies τ, variables may include types of technologies, scale and factor prices. First, the hedonic equation is estimated in the normal manner, without these variables, using the best-fitting functional form. This is to represent the price function that consumers and producers face when making their decisions. Then, an implicit marginal price function is computed for each characteristic as $\partial p(z)/\partial z_i = \hat{p}_i(z)$, where $\hat{p}(z)$ is the estimated hedonic equation. Bear in mind that in normal demand/supply studies for *products*, the prices are observed in the market. Prices for *characteristics* are unobserved; this first stage is to estimate the parameters from the hedonic regression. The actual values of each z_i bought and sold are then inserted into each implicit marginal price function to yield a numerical value for each characteristic. These marginal values are used in the second stage[51] of estimation as endogenous variables for the estimation

[50] These are equivalent to inverse demand (or supply) functions, with the prices dependent upon the quantities demanded (or supplied) and the individual consumer (or producer) characteristics.

of the demand side:

$$\hat{p}_i(z) = F(z_1,\ldots,z_K, \alpha^*) \qquad \text{(A21.1)}$$

where α^* are the proxy variables for tastes; and the supply side:

$$\hat{p}_i(z) = F(z_1,\ldots,z_K, \tau^*) \qquad \text{(A21.2)}$$

where τ^* are the proxy variables for technologies.

The variables τ^* drop out when there is no variation in technologies and $\hat{p}_i(z)$ is an estimate of the offer function. Similarly, the variables α^* drop out when sellers differ and buyers are identical, and cross-section estimates trace out compensated demand functions.

3. Epple (1987) has argued that Rosen's modelling strategy is likely to give rise to inappropriate estimation procedures of the demand and supply parameters. In the hedonic approach to estimating the demand for characteristics, a difficulty arises from the fact that marginal prices are likely to be endogenous – they depend on the amount of each characteristic consumed and must be estimated from the hedonic function rather than observed directly. There are two resulting problems. First, there is an identification problem (see Epple (1987)) because both the marginal price of a characteristic and the inverse bid depend on the levels of characteristics consumed. Second, if important characteristics are unmeasured and they are correlated with measured characteristics, the coefficients of measured characteristics will be biased. This applies to all econometric models, but is particularly relevant to hedonic models; on this point, see Wooldridge (1996, pp. 400-401) in particular. The equilibrium conditions for characteristic prices imply functional relationships among the characteristics of demanders, suppliers and products. This in turn reduces the likelihood that important excluded variables will be uncorrelated with the included variables of the model; see also Bartik (1988) on this point. The bias arises because buyers are differentiated by characteristics (y,α) and sellers by technologies τ. The type of item buyers will purchase is related to (y,α) and the type sellers provide to τ. On the plane of combinations of z transacted, the equilibrium ones chosen may be systematically related; the characteristics of buyers are related to those of sellers. Epple (1987) uses the example of stereo equipment: the higher income of some buyers leads to purchases of high-quality equipment, and the technical competence of sellers leads them to provide it. The consumer and producer characteristics may be correlated.

4. Wooldridge (1996, pp. 400-401) suggests that individual consumer and firm characteristics, such as income, education and input prices, should be used as instruments in estimating hedonic functions. In addition, variables other than a good's characteristics should be included as instruments if they are price-determining, such as geographical location (proximity to ports, good road systems, climate and so on). Communities of economic agents are assumed, within which consumers consume and producers produce for each other at prices that vary across communities for identical goods. Variables of the characteristics of the communities will not in themselves enter the demand and supply equation, but are price-determining for observed prices that are recorded across communities. Tauchen and Witte

[51] This two-stage approach is common in the literature, though Wooldridge (1996) discusses the joint estimation of the hedonic, demand-side and supply-side functions as a system.

(2001) provide a systematic investigation of the conditions under which the characteristics of consumers, producers and communities will affect the hedonic parameter estimates for a single regression equation estimated across all communities. A key concern is whether the error term of the hedonic price function represents factors that are unobserved by both the economic agents and the researcher, or only by the researcher. In the latter case, the error term may be correlated with the product attributes; instrumental variable estimation is required. If the error term is *not* correlated with the product characteristics – preferences are quasi-linear – then a properly specified hedonic regression, including community-specific characteristics or appropriate slope dummies, can be estimated using ordinary least squares. In other cases, depending on the correlation between consumer and producer characteristics, assumptions about the error term and the method of incorporating community characteristics into the regression, instrumental variables, including consumer, producer or community dummy or characteristics, may need to be used.

Functional form

5. Triplett (1987 and 2002) argues that neither classical utility theory nor production theory can specify the functional form of the hedonic function.[52] This point dates back to Rosen (1974, p. 54) who describes the observations as being "... a joint-envelope function and cannot by themselves identify the structure of consumer preferences and producer technologies that generate them". A priori judgements as to what the form should look like may be based on ideas as to how consumers and production technologies respond to price changes. These judgements are difficult to make when the observations are jointly determined by demand and supply factors, but not impossible in rare instances. They are, however, complicated when pricing is with a mark-up, the extent of which may vary over the life cycle of a product. Some tied combinations of characteristics will have higher mark-ups than others. New item introductions are likely to be attracted to these areas of characteristic space, and this will have the effect of increasing supply and thus lowering the mark-up and price; see Cockburn and Anis (1998), Feenstra (1995, p. 647) and Triplett (1987). This again must be taken into account in any a priori reasoning – not an easy or straightforward matter.

6. It may be that in some cases the hedonic function's functional form will be straightforward. For example, prices on the web sites for options for products are often additive. The underlying cost and utility structures are unlikely jointly to generate such linear functions, but the producer or consumer is also paying for the convenience of selling in this way and is willing to bear losses or make gains if the cost or utility at higher values of z are priced lower or worth more than the price set. In general, the data should convey what the functional form should look like; imposing artificial structures simply leads to specification bias. For examples of econometric testing of hedonic functional form, see Cassel and Mendelsohn (1985), Cropper, Deck and McConnell (1988), Rasmussen and Zuehlke (1990), Bode and van Dalen (2001) and Curry, Morgan and Silver (2001).

7. The three forms prevalent in the literature are linear, semi-logarithmic and double-

[52] Although Arguea, Haseo and Taylor (1994) propose a linear form on the basis of arbitrage for characteristics, held to be likely in competitive markets, Triplett (2002) argues that this is unlikely to be a realistic scenario in most commodity markets.

logarithmic (log-log). A number of studies have used econometric tests, in the absence of a clear theoretical statement, to choose between them. There have been a large number of hedonic studies and, as illustrated by Curry, Morgan and Silver (2001), in many of these the quite simple forms do well, at least in terms of the $\overline{R}^2$ presented[53] and the parameters according with a priori reasoning, usually on the consumer side. Of the three popular forms, some are favoured in testing; for example, Murray and Sarantis (1999) favoured the semi-logarithmic form, while others, for example Hoffmann (1998), found that the three functional forms scarcely differed in terms of their explanatory power. That the parameters from these simple forms accord with a priori reasoning, usually from the consumer side, is promising, but researchers should be aware that such matters are not assured. There is much that may happen on the supply side to affect parameter values. Indeed Pakes (2001) has argued that no intuitive sign can be given to the parameters of the variables, since producers may vary their price mark-ups on characteristics in ways that would result in counter-intuitive negative signs on some desirable characteristics.

8. Of the three forms, the semi-logarithmic form has much to commend it. The interpretation of its coefficients is quite straightforward, as proportionate changes in prices arise from a unit change in the value of the characteristic[54] (see Chapter 7, paragraphs 7.39 and 7.40). This is a useful formulation since quality adjustments are usually undertaken by making multiplicative as opposed to additive adjustments.

9. The semi-logarithmic form, unlike the log-log model, can incorporate dummy variables for characteristics which are either present, z_i=1, or not, z_i=0. Furthermore, Diewert (2002e) has argued that it is more likely that the errors from a semi-logarithmic hedonic equation are homoskedastic (have a constant variance) compared to the errors from a linear hedonic equation, since items with very large characteristic values will have high prices and are very likely to have relatively large error terms. On the other hand, models with very small amounts of characteristics will have small prices and small means, and the deviation of a model price from its mean will necessarily be small. Since an assumption of OLS is that the residuals are homoskedastic, the semi-logarithmic equation is preferred to the linear one.

10. More complicated forms are, of course, possible. Simple forms have the virtue of parsimony and allow more efficient estimates to be made for a given sample. However, parsimony is not something to be achieved at the cost of misspecification bias. First, if the hedonic function is estimated across multiple independent markets, then interaction terms are required (see Mendelsohn (1984) for fishing sites). Excluding them is tantamount to omitting variables and inappropriately constraining the estimated coefficients of the regression.

[53] While the use of $\overline{R}^2$ as a criterion for deciding between the fit of semi-logarithmic and log-log models has some validity, its use is not advised for comparing linear models with either of these logarithmic formulations, a number of tests being appropriate for such comparisons; see Maddala (1988).

[54] There are two caveats: first, $e^{\hat{\beta}-1}$ is required for the interpretation of the coefficients, where $\hat{\beta}$ is the estimated coefficient. Second, the anti-logarithm of the OLS estimated coefficients are not unbiased – the estimation of semi-logarithmic functions as transformed linear regressions requires an adjustment to provide minimum variance unbiased estimates of parameters of the conditional mean. A standard adjustment is to subtract half of the coefficient's squared standard error from the estimated coefficient; see Goldberger (1968) and Teekens and Koerts (1972).

Tauchen and Witte (2001) have outlined the particular biases that can arise for such omitted variables in hedonic studies. Second, it may be argued that the functional form should correspond to the aggregator for the index – linear for a Laspeyres index, logarithmic for a geometric Laspeyres index, translogarithmic for a Törnqvist index, and quadratic for a Fisher index (see Chapter 17). As Triplett (2002) notes, however, the purpose of estimating hedonic regressions is to adjust prices for quality differences; imposing a functional form on the data which is inconsistent with the data might create an error in the quality adjustment procedure. Yet, as Diewert (2003a) notes, flexible functional forms encompass these simple forms, the log-log form being a special case of the translog form given in equation (17.42) and the semi-log form being a special case of the semi-log quadratic form given in equation (17.49). If there are *a priori* reasons to expect interaction terms for specific characteristics, as illustrated in the example in paragraph 7.99, then these more general forms allow this. The theory of hedonic functions neither dictates the form of the hedonic form nor restricts it.

Changing tastes and technologies

11. The estimates of the coefficients from a hedonic regression may change over time. Some of this change will be attributed to sampling error, especially if multicollinearity is present, as discussed below. But in other cases it may be a genuine reflection of changes in tastes and technologies. If a subset of the estimated coefficients from a hedonic regression is to be used to make a quality adjustment to a non-comparable replacement price, then the use of estimated out-of-date coefficients from some previous period to adjust the prices of the new replacement model may be inappropriate. There is a need to update the indices as regularly as the changes demanded.[55] Estimating hedonic imputation indices is more complicated. Silver (1999), using a simple example, showed how the estimate of quality-adjusted price changes requires a reference basket of characteristics. This is apparent for the hedonic imputation indices in paragraphs 21.37 to 21.60, where separate indices using base and current period characteristics are estimated. A symmetric average of such indices is considered appropriate. A hedonic index based on a time dummy variable implicitly constrained the estimated coefficients from the base and current periods to be the same. Diewert (2003a) formalized the problem of choosing the reference characteristics when comparing prices over time, when the parameters of the hedonic function may themselves be changing over time. He found the results of hedonic indices *not* to be invariant to the choice of reference period characteristic vector set z. He considered the use of a sales-weighted average vector of characteristics, as proposed by Silver (1999), but he notes that over long time periods this may become unrepresentative.[56] Of course, if a chained formulation is used, the weighted averages of characteristics remain reasonably up to date, although chaining has its own pros and cons (see paragraph 17.44 to 17.49 of Chapter 17). A fixed base alternative noted by Diewert (2003a) is to use a Laspeyres type comparison with the base period parameter set, and a Paasche type current period index with the current period parameter set, and take the geometric mean of the two indices for reasons similar to those given in Chapter 15, paragraphs 15.18 to 15.32. The resulting Fisher type index is akin to a geometric mean of

[55] Adjusting the base versus the current period price entails different data demands, see Chapter 7, paragraph 7.49.

[56] Other averages may of course be proposed; for example, the needs of an index representative of the "typical" establishment would be better met by a trimmed mean or median.

Laspeyres and Paasche indices – given in equations (21.26) and (21.27) – based on Feenstra (1995).[57] A feature of the time dummy approach in paragraphs 21.40 to 21.42 is that it implicitly takes a symmetric average of the coefficients by constraining them to be the same. But what if, as is more likely the case, only base period hedonic regression coefficients are available? Since hedonic indices based on a symmetric average of the coefficients are desirable, the "spread" or difference between estimates based on either a current or a reference period characteristic set is an indication of potential bias and estimates of such spread may be undertaken retrospectively. If the spread is large, estimates based on the use of a single period's characteristics set, say the current period, should be treated with caution. More regular updating of the hedonic regressions is likely to reduce spread because the periods being compared will be closer and the characteristics of the items in the periods compared more similar.

Weighting

12. Ordinary least squares estimators implicitly treat each item as being of equal importance, even though some items will have quite substantial sales, while sales of others will be minimal. It is axiomatic that an item with sales of over 5,000 in a month should not be accorded the same influence in the regression estimator as one with a few transactions. Items with very low sales may be at the end of their life cycles or be custom-made. Either way, their (quality-adjusted) prices and price changes may be unusual.[58] Observations with unusual prices should not be allowed unduly to influence the index.[59]

13. The estimation of hedonic regression equations by a weighted least squares (WLS) estimator is preferable. This estimator minimizes the sum of *weighted* squared deviations between the actual prices and the predicted prices from the regression equation, as opposed to ordinary least squares (OLS), which uses an equal weight for each observation. There is a question as to whether to use quantity (volume) or expenditure weights. The use of quantity weights can be supported by considering the nature of their equivalent "price". Such prices are the average (usually the same) price over a number of transactions. The underlying sampling unit is the individual transaction, so there is a sense that the data may be replicated as being composed of, say, 12 individual observations using an OLS estimator, as opposed to a single observation with a weight of 12 using a WLS estimator. Both would yield the same result. Diewert (2002e) has argued on the grounds of representativity that sales values are the appropriate weights. Quantity weighting gives too little weight to models with high prices and too much weight to cheap models that have relatively low amounts of useful characteristics. The need to equate the weights with relative expenditure or sales value arises from a prime concern with index numbers: that they serve to decompose changes in value

[57] Diewert (2002e) also suggests matching items where possible, and using hedonic regressions to impute the prices of the missing old and new ones. Different forms of weighting systems, including superlative ones, can be applied to this set of price data in each period for both matched and unmatched data.

[58] Such observations would have higher variances of their error terms, leading to imprecise parameter estimates. This would argue for the use of weighted least squares estimators with quantity sold as the weight. This is one of the standard treatments for heteroskedastic errors; see Berndt (1991).

[59] Silver and Heravi (2002) show that old items have above-average leverage effects and below-average residuals. Not only are they different, but they exert undue influence for their size (number of observations). See Berndt, Ling and Kyle (2003), Cockburn and Anis (1998) and Silver and Heravi (2002) for examples.

into their price and quantity components. Silver (2002) has shown that a WLS estimator using value weights will not necessarily give each observation a weight equal to its relative value. The estimator will give more weight to those observations with high leverage effects and residuals. Observations with values of characteristics with large deviations from their means, say very old or new models, have relatively high leverage. New and old models are likely to be priced at quite different prices than those predicted from the hedonic regression, even after taking into account their different characteristics. Such prices result, for example, from a pricing strategy designed to skim segments of the market willing to pay a premium for a new model, or from a strategy to charge relatively low prices for an old model to dump it to make way for a new one. In such cases, the influence these models have on deriving the estimated coefficients will be over and above that attributable to their value weights. Silver (2002) suggests that leverage effects should be calculated for each observation, and those with high leverage and low weights should be deleted, and the regression rerun. Thus, while quantity or value weights are preferable to no weights (i.e., OLS), value weights are more appropriate than quantity ones and, even so, account should be taken of observations with undue influence.

14. Diewert (2002e) has also considered the issue of weighting with respect to the dummy time hedonic indices outlined in paragraphs 21.40 to 21.42. The use of WLS by value involves weights being applied to observations in both periods. However, if, for example, there is high inflation then the sales values for a model in the current period will generally be larger than those of the corresponding model in the base period and the assumption of homoskedastic residuals is unlikely to be met. Diewert (2002e) suggests the use of expenditure *shares* in each period, as opposed to values, as weights for WLS for time dummy hedonic indices. He also suggests that an average of expenditure shares in the periods being compared be used for matched models.

15. Data on sales are not always available for weights, but the major selling items can generally be identified. In such cases, it is important to restrict the number of observations of items with relatively low sales, the extent of the restriction depending on the number of observations and the skewness of the sales distribution. In some cases, items with few sales provide the variability necessary for efficient estimates of the regression equation. In other cases, their low sales may be due to factors that make them unrepresentative of the hedonic surface, their residuals being unusually high. An example is low-selling models about to be dumped to make way for new models. Unweighted regressions may thus suffer from a sampling problem – even if the prices are perfectly quality adjusted, the index can be biased because it is unduly influenced by low-selling items with unrepresentative price–characteristic relationships. In the absence of weights, regression diagnostics have a role to play in helping to determine whether the undue variance in some observations belongs to such unusual low-selling items.[60]

[60] A less formal procedure is to take the standardized residuals from the regression and plot them against model characteristics that may denote low sales, such as certain brands (makes) or vintage (if not directly incorporated), or some technical feature which makes it unlikely that the item is being bought in quantity. Higher variances may be apparent from the scatter plot. If certain features are expected to have, on average, low sales, but seem to have high variances, leverages and residuals (see Silver and Heravi (2002)), a case exists for at least down-playing their influence. Bode and van Dalen (2001) use formal statistical criteria to decide between different weighting systems and compare the results of OLS and WLS, finding, as with Ioannidis and

Multicollinearity

16. There are a priori reasons to expect, for some products, that the variation in the value of one quality characteristic is not independent of one quality characteristic or a linear combination of more than one such characteristic. As a result, parameter estimates will be unbiased yet imprecise. To illustrate this, a plot of the confidence interval for one parameter estimate against another collinear one is often described as elliptical, since the combinations of possible values they may take can easily drift from, say, high values of β_1 and low values of β_2 to high values of β_2 and low values of β_1. Since the sample size for the estimates is effectively reduced, additions to and deletions from the sample may affect the parameter estimates more than would be expected. These are standard statistical issues, dealt with by Maddala (1988) and Kennedy (1998). In a hedonic regression, multicollinearity might be expected, as some characteristics may be technologically tied to others. Producers including one characteristic may need to include others for the product to work, while consumer purchasing, say, an up-market brand may expect a certain bundle of features to come with it. Triplett (2002) argues strongly for the researcher to be aware of the features of the product and the consumer market. There are standard, though not completely reliable, indicators of multicollinearity (such as variance inflation factors), but an exploration of its nature is greatly aided by an understanding of the market along with exploration of the effects of including and excluding individual variables on the signs and coefficients and on other diagnostic test statistics; see Maddala (1988).[61]

17. If a subset of the estimated coefficients from a hedonic regression is to be used to quality-adjust a non-comparable replacement price, and if there is multicollinearity between variables in this subset and other independent variables, then the estimates of the coefficients to be used for the adjustment will be imprecise. The multicollinearity effectively reduces the sample size, and some of the effects of the variables in the subset may be wrongly ascribed to the other independent variables. The extent of this error will be determined by the strength of the multiple correlation coefficient between all such "independent" variables (the multicollinearity), the standard error or fit of the regression, the dispersion of the independent variable concerned and the sample size. These all affect the precision of the estimates since they are components in the standard error of the *t*-statistics. Even if multicollinearity is expected to be quite high, large sample sizes and a well-fitting model may reduce the standard errors on the *t*-statistics to acceptable levels. If multicollinearity is expected to be severe, the predicted value for an item's price may be computed using the whole regression and an adjustment made using this predicted value, as explained in Chapter 17, paragraphs 17.103 to 17.109. There is a sense in which it does not matter whether the variation that, for example, should have been attributed to β_1 was wrongly attributed to β_2, or vice versa if the predicted price based on both β_1 and β_2 is used.

Omitted variable bias

Silver (1999), that different results can arise.

[61] Triplett (2002) stresses the point that $\bar{R}^2$ alone is insufficient for this purpose.

18. The exclusion of tastes, technology and community characteristics has already been discussed. The concern here is with product characteristics. Consider again the use of a subset of the estimated coefficients from a hedonic regression to quality-adjust a non-comparable replacement price. It is well established that multicollinearity of omitted variables with included variables leads to bias in the estimates of the coefficients of included ones. If omitted variables are independent of the included variables, then the estimates of the coefficients on the included variables are unbiased. This is acceptable in the present instance, the only caveat being that the quality adjustment for the replacement item may also require an adjustment for these omitted variables and this, as noted by Triplett (2002), has to be undertaken using a separate method and data. But what if the omitted variable is multicollinear with a subset of included variables which are to be used to quality-adjust a non-comparable item? In this case, the coefficient of the subset of the included variables may wrongly pick up some of the effects of the omitted variables. The subset of included variables will be used to quality-adjust prices for items which differ only with regard to this subset, and the price comparison will be biased if the characteristics of included and omitted variables have different price changes. For hedonic indices using a dummy time trend, the estimates of quality-adjusted price changes will suffer from a similar bias if omitted variables that are multicollinear with the time change are excluded from the regression. What are picked up as quality-adjusted price changes over time may, in part, be changes attributable to the prices of these excluded variables. This happens when the prices of the omitted characteristics follow a different trend. Such effects are most likely when there are gradual improvements in the quality of items, such as the reliability and safety of consumer durables,[62] which are difficult to measure, at least for the sample of items in real time. The quality-adjusted price changes will thus overstate price changes in such instances.

[62] There are, of course, some commodity areas, such as airline comfort, which have been argued to have overall patterns of decreasing quality.